Key

4.1 # 4, 8, 12, 4.2 # 4, 12, 28, 4.3 # 6, 10, 14, 18, 24, 28, 26
4.4 # 2, 6, 24, 32 4.5 # 6, 18, 48, 60, 78 4.6 # 12, 22, 24, 30

Section 4.1

4) a) $\dfrac{5^{3}}{5^{6}} = 5^{3-6} = 5^{-3} = \dfrac{1}{5^{3}} = \dfrac{1}{125}$

b) $\left(\dfrac{1}{5}\right)^{-2} = \dfrac{1}{(1/5)^{2}} = \dfrac{1}{1/5^{2}} = 5^{2} = 25$

c) $(8^{1/2})(2^{1/2}) = (8\times 2)^{1/2} = 16^{1/2} = \sqrt{16} = 4$

d) $(32^{3/2})\left(\dfrac{1}{2}\right)^{3/2} = \left(32\times\dfrac{1}{2}\right)^{3/2} = 16^{3/2} = (16^{1/2})^{3} = 4^{3} = 64$

8) $f(x) = 3^{x+2}$

a) $f(-4) = 3^{-4+2} = 3^{-2} = \dfrac{1}{3^{2}} = \dfrac{1}{9}$

b) $f(-\tfrac{1}{2}) = 3^{-1/2+2} = 3^{3/2} \approx 5.196$

c) $f(2) = 3^{2+2} = 3^{4} = 81$

d) $f(-5/2) = 3^{-5/2+2} = 3^{-1/2} = \dfrac{1}{\sqrt{3}} \approx 0.577$

12) $P(t) = 252.12(1.011)^{t}$ $t=2$ is 1992

$\Rightarrow t=10$ is 2000

a) $2008 \to t=18$

$P(18) = 252.12(1.011)^{18}$

≈ 306.993 million people

b) $2012 \to t=22$

$P(22) = 252.12(1.011)^{22}$

≈ 320.725 million people

Section 4.2

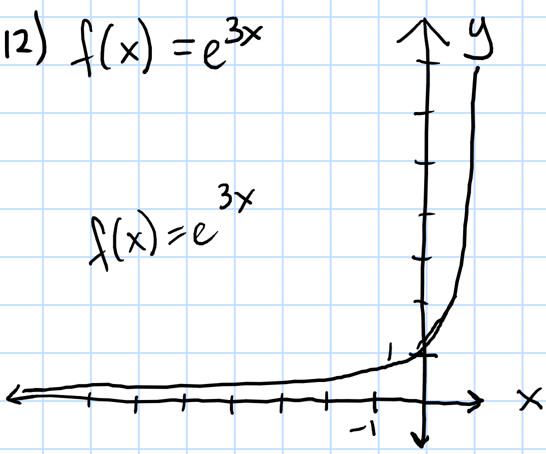
4) a) $(e^{-3})^{2/3} = e^{-3(2/3)} = e^{-2} = \dfrac{1}{e^{2}}$

b) $\dfrac{e^{4}}{e^{-1/2}} = e^{4-(-1/2)} = e^{4+1/2} = e^{9/2}$

c) $(e^{-2})^{-4} = e^{-2(-4)} = e^{8}$

d) $(e^{-4})(e^{-3/2}) = e^{-4-3/2} = e^{-8/2-3/2} = e^{-11/2} = \dfrac{1}{e^{11/2}}$

12) $f(x) = e^{3x}$



28) $y = 925/(1+e^{-0.3t})$

a) Graphing utility

b) Yes – it appears to have a maximum at 925. (based on graph)

c) The limit would increase to 1000.

Note: $\lim_{t\to\infty} e^{-0.3t} = 0$

and $\lim_{t\to\infty} \frac{925}{1+e^{-0.3t}} = \frac{925}{1+0} = 925$

These models reflect some kind of max population for an ecosystem (or petri dish).

Section 4.3

6) $y = e^{1-x}$ let $u = 1-x$

$y' = e^u \frac{du}{dx}$ $\frac{du}{dx} = -1$

$= e^{1-x}(-1)$

$= -e^{1-x}$

10) $g(x) = e^{\sqrt{x}}$ let $u = \sqrt{x} = x^{1/2}$

$g'(x) = e^{x^{1/2}}\left(\frac{1}{2}x^{-1/2}\right)$ $\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

14) $f(x) = \frac{(e^x + e^{-x})^4}{2}$

$f'(x) = \frac{1}{2}\frac{d}{dx}\left[(e^x + e^{-x})^4\right]$

$= \frac{1}{2}\left[4(e^x+e^{-x})^3 \frac{d}{dx}(e^x + e^{-x})\right]$ Chain rule

$= 2(e^x+e^{-x})^3(e^x - e^{-x})$ $u = -x$ $\frac{du}{dx} = -1$

18) $g(x) = e^{x^3}$ at $\left(-1, \frac{1}{e}\right)$

let $u = x^3$ $du = 3x^2$

$g'(x) = e^{x^3}(3x^2) = 3x^2e^{x^3}$

$g'(-1) = 3(-1)^2e^{(-1)^3} = 3e^{-1} = \frac{3}{e}$

Point-slope formula:

$y - y_0 = m(x - x_0)$

$y - \frac{1}{e} = \frac{3}{e}[x - (-1)]$

$y - \frac{1}{e} = \frac{3}{e}(x+1)$

$y = \frac{3x}{e} + \frac{3}{e} + \frac{1}{e}$

$= \frac{3}{e}x + \frac{4}{e}$

24) $f(x) = (1+2x)e^{4x}$

$f'(x) = (1+2x)\frac{d}{dx}[e^{4x}] + 2e^{4x}$ let $u = 4x$

$= (1+2x)(4e^{4x}) + 2e^{4x}$ $\frac{du}{dx} = 4$

$= 2e^{4x}(2)(1+2x) + 2e^{4x}$

$= 2e^{4x}(3+2x)$

$f''(x) = 2e^{4x}(2) + (3+2x)\frac{d}{dx}[2e^{4x}]$ let $u = 4x$

$= 4e^{4x} + (3+2x)(8e^{4x})$

$= 4e^{4x}(1+6+4x)$

$= 4e^{4x}(7+4x)$

28) $f(x) = \frac{e^x - e^{-x}}{2}$ Graph in Desmos.

$f'(x) = \frac{1}{2}(e^x + e^{-x})$ $u = -x$ $\frac{du}{dx} = -1$ no asymptotes

never $=0$ or undefined!

$f''(x) = \frac{1}{2}(e^x - e^{-x}) = f(x)$

$=0$ when $e^x = e^{-x}$ at $x = 0$

Interval	Sign of $f''(x)$	Conclusion
$(-\infty, 0)$	$-$	Concave down
$(0, \infty)$	$+$	Concave up

point of inflection at $(0,0)$

46) The mean shifts the function horizontally along the x-axis. This has no impact on the shape.

Section 4.4

2) $\ln 9 = 2.1972\ldots$

$e^{\ln 9} = e^{2.1972\ldots}$

$e^{2.1972\ldots} = 9$

6) $e^2 = 7.3891\ldots$

$\ln(e^2) = \ln(7.3891\ldots)$

$\ln(7.3891\ldots) = 2$

24) $\ln e^{2x-1} = 2x-1$

32) $\ln(\frac{1}{5}) = \ln(1) - \ln(5)$

$= 0 - \ln(5)$

$= -\ln(5)$

Section 4.5

6) $f(x) = \ln(2x)$ $u = 2x$ $\frac{du}{dx} = 2$

$f'(x) = \frac{1}{u}\frac{du}{dx}$

$= \frac{2}{2x} = \frac{1}{x}$

18) $y = \ln\left(\frac{x^2}{x^2+1}\right)$ $u = \frac{x^2}{x^2+1}$ $\frac{du}{dx} = \frac{(x^2+1)(2x) - x^2(2x)}{(x^2+1)^2}$

$= \frac{2x}{(x^2+1)^2}$

$y' = \frac{1}{x^2/(x^2+1)}\left(\frac{2x}{(x^2+1)^2}\right) = \frac{\cancel{x^2+1}}{x^{\cancel{2}}}\left(\frac{\cancel{2x}}{(x^2+1)^{\cancel{2}}}\right)$

$= \frac{2}{x(x^2+1)}$

48) $y = x3^{x+1}$

$y' = x\frac{d}{dx}[3^{x+1}] + 3^{x+1}(1)$ let $u = x+1$ $a=3$

$= x[\ln(3)(3^{x+1})(1)] + 3^{x+1}$ $\frac{du}{dx} = 1$

$= x\ln 3(3^{x+1}) + 3^{x+1}$

$= 3^{x+1}(x\ln 3 + 1)$

60) $T = 87.97 + 34.96\ln p + 7.91\sqrt{p}$

$T' = \frac{34.96}{p} + 7.91\left(\frac{1}{2}p^{-1/2}\right)$

$= \frac{34.96}{p} + \frac{3.955}{\sqrt{p}}$

At $p=60$,

$T' = \frac{34.96}{60} + \frac{3.955}{\sqrt{60}} \approx 1.093$

78) $R = \frac{\ln I - \ln I_0}{\ln 10}$ assume $I_0 = 1 \rightarrow \ln(1) = 0$

$= \frac{\ln I}{\ln 10}$ for $I_0 = 1$

$R(\ln 10) = \ln I$

$\ln(10^R) = \ln I$

$e^{\ln(10^R)} = e^{\ln I}$

$I = 10^R$

a) $R = 8.3 \longrightarrow I = 10^{8.3} \approx 1.995 \times 10^8$

b) $R = 6.3 \longrightarrow I = 10^{6.3} \approx 1.995 \times 10^6$

c) If R is doubled,

$I = 10^R$

$I_2 = 10^{2R}$

$= (10^R)^2$

$= I^2$

$= I \cdot I$

Increases by a factor of I

d) $\frac{dR}{dI} = \frac{d}{dI}\left[\frac{\ln I}{\ln 10}\right] = \frac{1}{\ln 10}\frac{d}{dI}(\ln I)$

$= \frac{1}{\ln 10}\left(\frac{1}{I}\right)$

$= \frac{1}{I\ln 10}$

Section 4.6

12) $\frac{dy}{dt} = -\frac{2}{3}y$ $y = 20$ when $t = 0$

$y = Ce^{kt}$

$20 = Ce^{k(0)}$

$C = 20$

$\frac{dy}{dt} = Cke^{kt} = -\frac{2}{3}y = -\frac{2}{3}(Ce^{kt})$

$y = 20e^{-2/3 t}$

$\rightarrow kCe^{kt} = -\frac{2}{3}Ce^{kt}$

exponential decay

So $k = -2/3$

22) At time 0, 100% time in years
time 1, 99.57%

100% $\to$ 1 99.57% $\to$ 0.9957

$1 = Ce^{k(0)}$ $1 = C$
$0.9957 = Ce^{k(1)}$
$= e^{k}$
$\ell(0.9957) = k$
$y = e^{\ell(0.9957)t}$

half-life = time at which 50% remains.

$0.5 = e^{\ell(0.9957)t}$
$\ell(0.5) = \ell(0.9957)t$
$t = \frac{\ell(0.5)}{\ell(0.9957)} \approx 160.85$ years

24) Half life is 5715 years.

$\to 0.5 = Ce^{5715k}$

At time 0, 100% remains

$\to 1 = Ce^{0} \to C = 1$

$0.5 = e^{5715k}$
$\ell(0.5) = 5715k$
$k \approx -0.000121$

$\Big\} \; y = e^{-0.000121t}$

Charcoal at 30% carbon:

$0.3 = e^{-0.000121t}$
$\ell(0.3) = -0.000121t$
$t \approx 9950.188$ years

30) $N = 100e^{kt}$ $N = 300$ when $t = 5$

$300 = 100e^{k(5)}$
$3 = e^{5k}$
$\ell(3) = 5k$
$k = 0.2197$

$N = 100e^{0.2197t}$

Double (from 100 to 200)

$200 = 100e^{0.2197t}$
$2 = e^{0.2197t}$
$\ell(2) = 0.2197t$

$t = 3.155$ hours