Section 1.4

3. Does the equation define y as a function of x?

$$\frac{1}{2}x - 6y = -3$$

$$-6y = -\frac{1}{2}x - 3$$

$$y = \frac{1}{12}x + \frac{1}{2}$$ linear

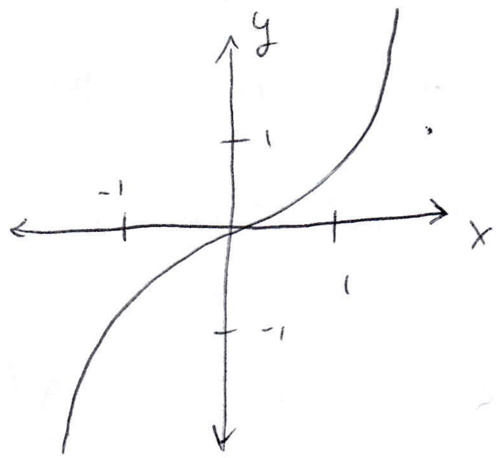
So this is a function.

17. Find the domain and range.

$$f(x) = x^3$$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$



25. $f(x) = x^2 - 5x + 2$

Evaluate

$$\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

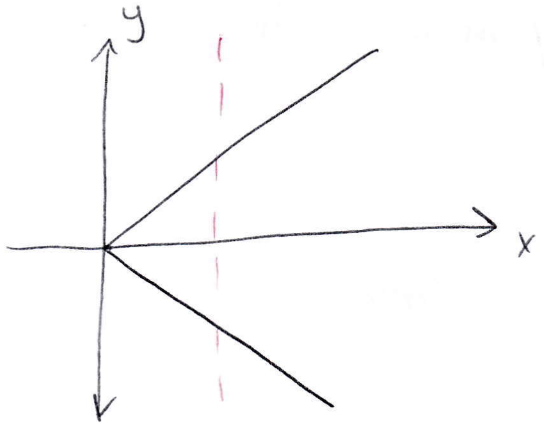
$$\frac{\left[(x+\Delta x)^2 + - 5(x+\Delta x) + 2\right] - (x^2 - 5x + 2)}{\Delta x}$$

$$= \frac{\cancel{x^2} + 2x\Delta x + (\Delta x)^2 - \cancel{5x} - 5\Delta x \cancel{+2} - \cancel{x^2} \cancel{+5x} \cancel{-2}}{\Delta x}$$

$$= \frac{2x\cancel{\Delta x} + (\Delta x)^2 - 5\cancel{\Delta x}}{\cancel{\Delta x}} = 2x + \Delta x - 5$$

34. Use vertical line test to determine if y is a function of x.

$x = |y|$



y is NOT a function of x.

37. find (a) $f(x) + g(x)$ (b) $f(x)g(x)$ (c) $f(x)/g(x)$
(d) $f(g(x))$ (e) $g(f(x))$

$f(x) = x^2 + 1$ $g(x) = x - 1$

a) $f(x) + g(x) = (x^2+1) + (x-1) = x^2 + x + 1 - 1 = x^2 + x = x(x+1)$

b) $f(x)g(x) = (x^2+1)(x-1) = x^3 - x^2 + x - 1$

c) $f(x)/g(x) = \dfrac{x^2+1}{x-1}$

d) $f(g(x)) = [g(x)]^2 + 1 = (x-1)^2 + 1 = x^2 - 2x + 1 + 1$
$= x^2 - 2x + 2$

e) $g(f(x)) = [f(x)] - 1 = x^2 + 1 - 1 = x^2$

$f \circ g(x)$ $g \circ f(x)$

45. Find the inverse function.

$f(x) = 2x - 3$

$y = 2x - 3$

$y + 3 = 2x$

$\frac{1}{2}y + \frac{3}{2} = x$

$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$

$\boxed{\text{Section 1.5}}$

23. Find the limit $\lim_{x \to 2} x^2$

This is a polynomial $\rightarrow$ plug in $x = 2$

$\lim_{x \to 2} x^2 = 2^2 = 4$

37. $\lim_{x \to 3} \frac{\sqrt{x+1} - 1}{x}$ quotient operation $\frac{\lim_{x \to 3} \sqrt{x+1} - 1}{\lim_{x \to 3} x}$

as long as this is nonzero

Consider $\lim_{x \to 3} x = 3$

Consider $\lim_{x \to 3} \sqrt{x+1} - 1$ addition operation $\lim_{x \to 3} \sqrt{x+1} - \underbrace{\lim_{x \to 3} 1}_{1}$

$\lim_{x \to 3} \sqrt[2]{x+1}$ radical operation $\sqrt[2]{\lim_{x \to 3} (x+1)}$

(even root) limit must be positive

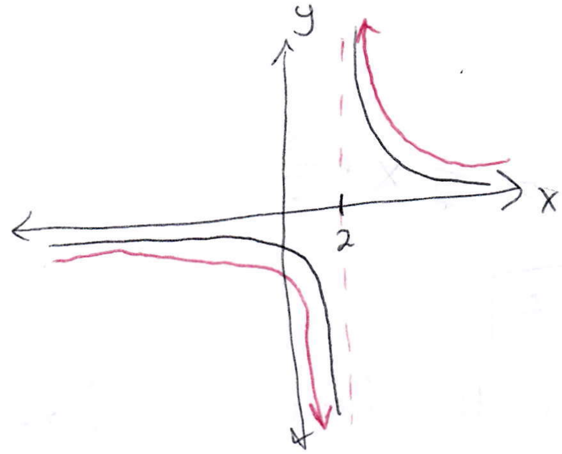
$= \sqrt{4} = 2$

$\lim_{x \to 3} \frac{\sqrt{x+1} - 1}{x} = \frac{2-1}{3} = \frac{1}{3}$

Ex. 43. Find the limit

$$\lim_{x \to 2} \frac{x-2}{x^2-4x+4}$$

$\dfrac{x-2}{x^2-4x+4}$ has domain $(-\infty, 2) \cup (2, \infty)$



$$\frac{\cancel{x-2}}{\cancel{(x-2)}(x-2)} = \frac{1}{x-2}$$

$$\lim_{x \to 2} \frac{1}{x-2}$$

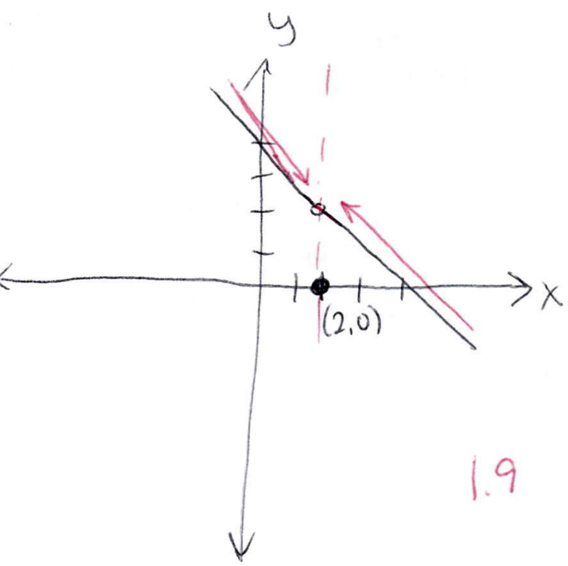
Consider

$$\lim_{x \to 2^-} \frac{1}{x-2} = -\infty$$

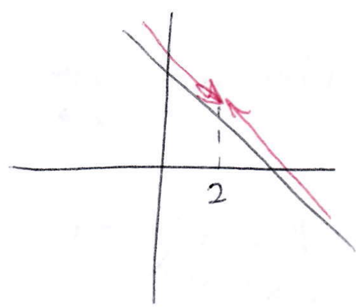
limit does not exist!

$$\lim_{x \to 2^+} \frac{1}{x-2} = \infty$$

51. $\lim\limits_{x \to 2} f(x)$ $\qquad f(x) = \begin{cases} 4-x & x \neq 2 \\ 0 & x = 2 \end{cases}$



$$g(x) = 4-x$$



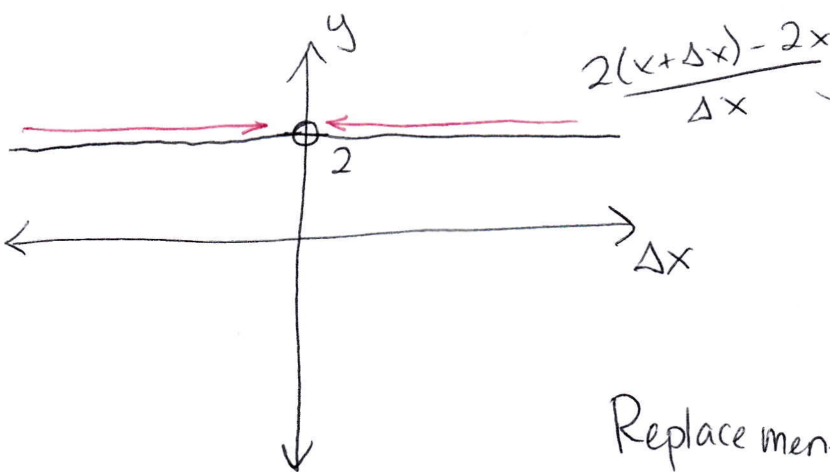
$$\lim_{x \to 2} g(x) = 4-2 = 2$$

1.9 1.99 1.9999999

55. $\lim\limits_{\Delta x \to 0} \left(\dfrac{2(x+\Delta x) - 2x}{\Delta x} \right)$

$\dfrac{2(x+\Delta x) - 2x}{\Delta x}$ domain: $\Delta x \neq 0$

$= \dfrac{2x + 2\Delta x - 2x}{\Delta x} = \dfrac{2\Delta x}{\Delta x} = 2$ for $\Delta x \neq 0$



Replacement theorem: $g(x) = 2$

$\lim\limits_{\Delta x \to 0} \left(\dfrac{2(x+\Delta x) - 2x}{\Delta x} \right) = 2$