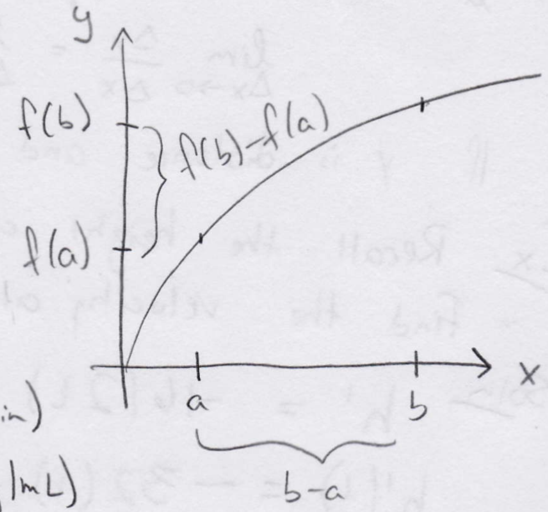
Rates of Change

Def Average rate of change

If $y = f(x)$, the average rate of change of y with respect to x on the interval $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a} = \frac{\Delta y}{\Delta x}$$



Ex The concentration C of a drug in a patient's bloodstream over an hour:

t	0	10	20	30	40	50	60	(min)
C	0	2	17	37	55	73	89	(mg/mL)

Find the avg. rate of change over $[0, 10]$ and $[0, 20]$.

Soln a) $\dfrac{\Delta C}{\Delta t} = \dfrac{2-0}{10-0} = \dfrac{1}{5}$ mg/mL/min

b) $\dfrac{\Delta C}{\Delta t} = \dfrac{17-0}{20-0} = \dfrac{17}{20} = 0.85$ mg/mL/min

Average Velocity:

$$\text{avg. velocity} = \frac{\text{change in distance}}{\text{change in time}} = \frac{\Delta d}{\Delta t}$$

Ex The height of a free-falling object dropped from a height of 100 feet is $h = -16t^2 + 100$

where h = height and t = time (sec). Find avg. velocity over $[1, 1.5]$.

Soln At $t = 1$, $h = -16 + 100 = 84$

$t = 1.5$, $h = -16(1.5)^2 + 100 = 64$

So $\dfrac{\Delta h}{\Delta t} = \dfrac{84-64}{1-1.5} = \dfrac{20}{-0.5} = -40$ ft/sec.

Instantaneous Rate of Change

Def: The instantaneous rate of change of $y = f(x)$ at x is the limit of the average rate of change on $[x, x+\Delta x]$ as $\Delta x \to 0$.

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

If y is distance and x is time, this is the velocity.

Ex Recall the height of a falling object $h = -16t^2 + 100$.
Find the velocity at $t = 1$.

Soln $h' = -16(2t) = -32t$

$h'(1) = -32(1) = -32$ ft/sec

The general position function for a free falling object is

$$h = -16t^2 + V_0 t + h_0$$

height (feet)

time (seconds)

initial velocity (ft/sec)

initial height

Note The absolute value of velocity is <u>speed</u>.

Ex At time $t = 0$, a diver jumps from a 32 ft diving board. Their initial velocity is 16 ft/sec. The <u>position function</u> is

$$h = -16t^2 + 16t + 32$$

When does the diver hit the water?

At $h = 0 \to$ $0 = -16t^2 + 16t + 32$

$= -16(t^2 - t - 2)$

$= -16(t-2)(t+1)$

$t = 2, \quad t = -1$

So they hit the water at $t = 2$ seconds.

b) What is their velocity at impact?

$h(t) = -16t^2 + 16t + 32$

The velocity is

$h'(t) = -32t + 16$

Impact occurs at $t = 2$.

$h'(2) = -32(2) + 16$
$= -48$ ft/sec.

Ex A holstein Calf's growth rate over one year:

t (months)	0	1	2	4	6	12
w (pounds)	96	118	161	272	396	714

This can be modeled using $w = 0.48t^2 + 47.3t + 80 \quad 0 \le t \le 12$

a) Find the avg. rate of change over the first year.

$$\frac{\Delta w}{\Delta t} = \frac{714-96}{12-0} = \frac{618}{12} = 51.5 \text{ lbs/mo}$$

b) Find the rate of change at month 4.

The rate of change is

$w' = 0.48(2t) + 47.3$
$= 0.96t + 47.3$

At $t = 4$

$w'(4) = 0.96(4) + 47.3$
$= 51.14$ lbs/mo