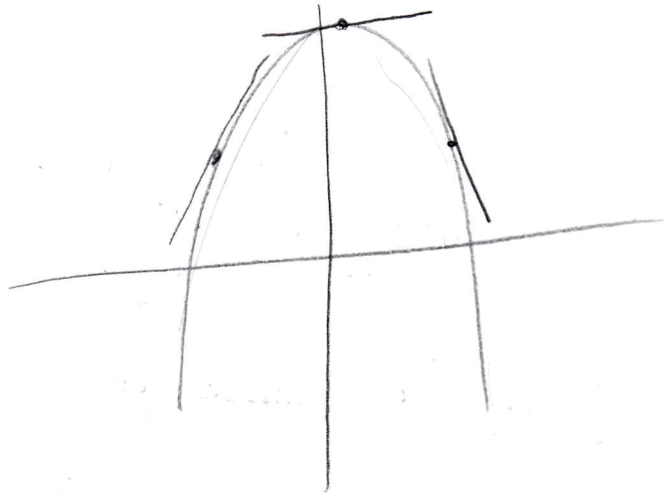
3.1 exercises

10) $f(x) = 5 - 3x$

$f'(x) = -3 \stackrel{?}{=} 0$

No value of x will make this zero!

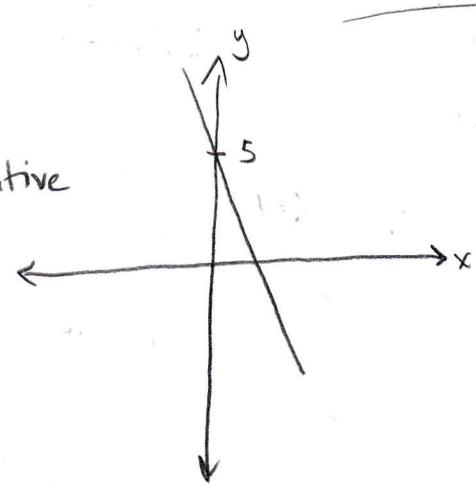
→No critical numbers!

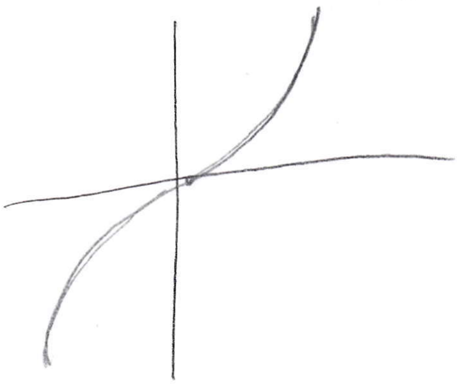


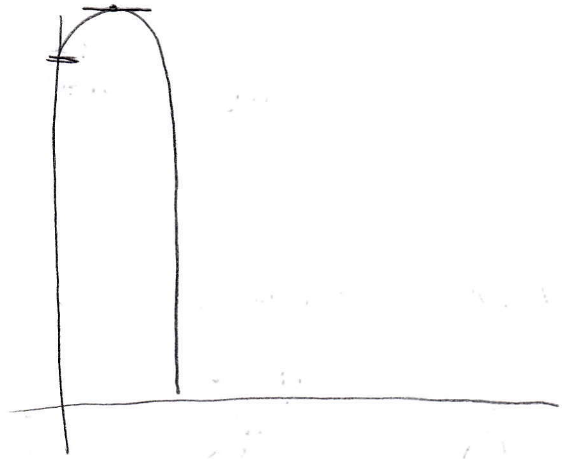
Interval

$(-\infty, \infty)$ slope is negative

decreasing function!







13) $y = x^2 - 6x$

$y' = 2x - 6$

$0 = 2x - 6$

$6 = 2x$

$3 = x$ critical number

Intervals	Test	Sign	Conclusion
$(-\infty, 3)$	0	$y' = -6 < 0$	decreasing
$(3, \infty)$	4	$y' = 2 > 0$	increasing

19) $y = x^{1/3} + 1$

$y' = \frac{1}{3}x^{-2/3}$

$0 = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$

$x = 0$ critical number

Intervals	Test	Sign	Conclusion
$(-\infty, 0)$	-1	+	increasing
$(0, \infty)$	1	+	increasing

y is increasing on $(-\infty, 0) \cup (0, \infty)$

discontinuity at $x=0$

$(-1)^{2/3} = \left[(-1)^2\right]^{1/3}$

31) $f(x) = \dfrac{x}{x^2+4} = x(x^2+4)^{-1}$

$$f'(x) = \frac{d}{dx}[x](x^2+4)^{-1} + \frac{d}{dx}\left[(x^2+4)^{-1}\right](x)$$
$$= (x^2+4)^{-1} + x\left[-1(x^2+4)^{-2}(2x)\right]$$
$$= \frac{(x^2+4)}{(x^2+4)^2} - \frac{2x^2}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}$$

$f'(x) = 0$ when $4 - x^2 = 0$

$(2-x)(2+x) = 0$

$x = \pm 2$

Intervals	Test	Sign	Conclusion
$(-\infty, -2)$	-3	$-$	decrease
$(-2, 2)$	0	$+$	increase
$(2, \infty)$	3	$-$	decrease

33) $f(x) = \dfrac{2x}{16-x^2} = 2x(16-x^2)^{-1}$

$$f'(x) = 2(16-x^2)^{-1} + \left[-1(16-x^2)^{-2}(-2x)\right](2x)$$
$$= \frac{2}{16-x^2} + \frac{4x^2}{(16-x^2)^2}$$
$$= \frac{2(16-x^2) + 4x^2}{(16-x^2)^2}$$
$$= \frac{2(x^2+16)}{(16-x^2)^2}$$

Intervals	Test	Sign	Concl.
$(-\infty, -4)$	-5	$+$	Increase
$(-4, 4)$	0	$+$	Increase
$(4, \infty)$	5	$+$	Increase

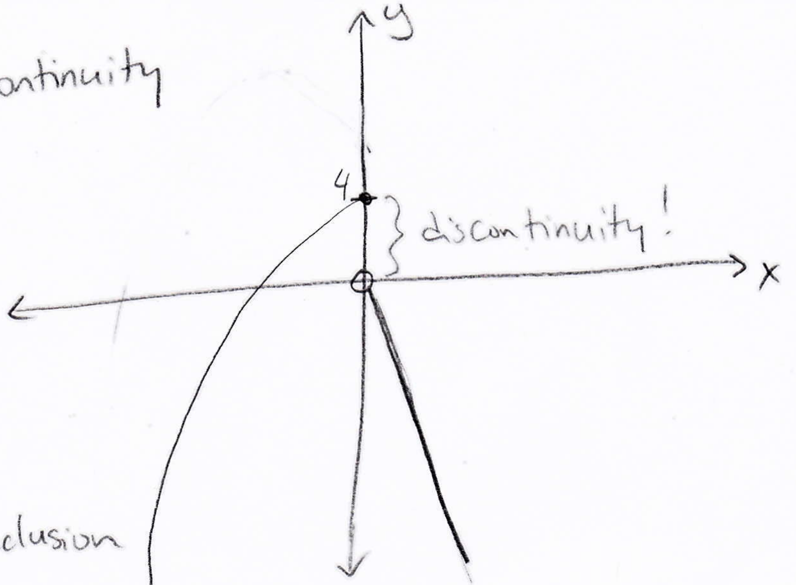
discontinuity at $x = \pm 4$

$2(x^2+16) = 0$

can never be 0!

$$y = \begin{cases} 4 - x^2 & x \le 0 \\ -2x & x > 0 \end{cases}$$

discontinuity



$$y' = \begin{cases} -2x & x \le 0 \\ -2 & x > 0 \end{cases}$$

$y' = 0$ at $x = 0$

Int.	Test	Sign	Conclusion
$(-\infty, 0)$	-1	$2 > 0$	Increasing
$(0, \infty)$	1	$-2 < 0$	Decreasing