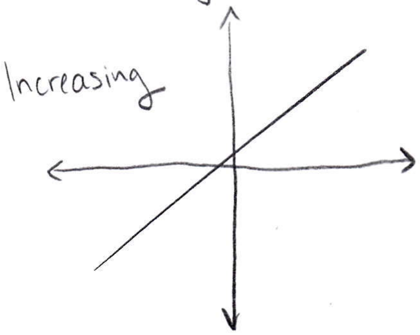
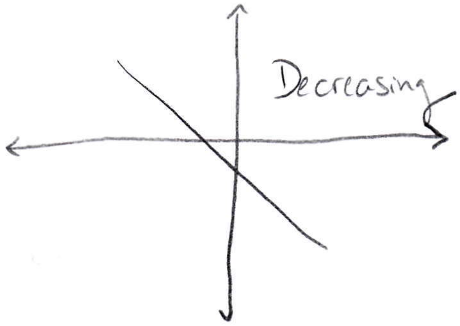
3.1 Increasing and Decreasing Functions





Def A function f is <u>increasing</u> on an interval if for any two numbers x_1 and x_2 in the interval,

$$x_2 > x_1 \quad \text{implies} \quad f(x_2) > f(x_1)$$

It is <u>decreasing</u> if

$$x_2 > x_1 \quad \text{implies} \quad f(x_2) < f(x_1).$$

Test for increasing / decreasing functions:

Let f be differentiable on the interval (a,b).

1) If $f'(x) > 0$ for all x in (a,b), then f is increasing on (a,b)
2) If $f'(x) < 0$ for all x in (a,b), then f is decreasing on (a,b).
3) If $f'(x) = 0$ for all x in (a,b), then f is Continuous on (a,b).

Ex Show that $f(x) = x^2$ is decreasing on $(-\infty, 0]$ and increasing on $[0, \infty)$.

Soln $f'(x) = 2x$.

$2x > 0$ when $x > 0$

$2x < 0$ when $x < 0$

Critical Numbers and Their Use

Consider $f(x) = x^2$. Where is it increasing/ decreasing?

Def If f is defined at c, then c is a critical number of f if $f'(c) = 0$ or if $f'(c)$ is undefined.

Determining Intervals of Increase / Decrease on f:

1) Find $f'(x)$.
2) Find the critical numbers of f.
3) Test the sign of $f'(x)$ at an arbitrary number chosen from each interval.
4) Use the test for increasing / decreasing functions to determine whether f is increasing / decreasing on each interval.

Ex $f(x) = (x^2-4)^{2/3}$

Soln
$$f'(x) = \frac{2}{3}(x^2-4)^{-1/3}(2x)$$
$$= \frac{4x}{3(x^2-4)^{1/3}}$$

$f'(x) = 0$ when $x = 0$. It is undefined for $x = \pm 2$.

Interval	Test	Sign		Conclusion
$(-\infty, -2)$	$x = -3$	$-/+$	negative	decreasing
$(-2, 0)$	$x = -1$	$-/-$	positive	increasing
$(0, 2)$	$x = 1$	$+/-$	negative	decreasing
$(2, \infty)$	$x = 3$	$+/+$	positive	increasing

Note
$$\frac{4(1)}{3(1-4)^{1/3}} = \frac{\text{positive}}{\text{negative}} = +/- = \text{negative}$$