Relative Extrema

Def: Let f be a function defined at c.

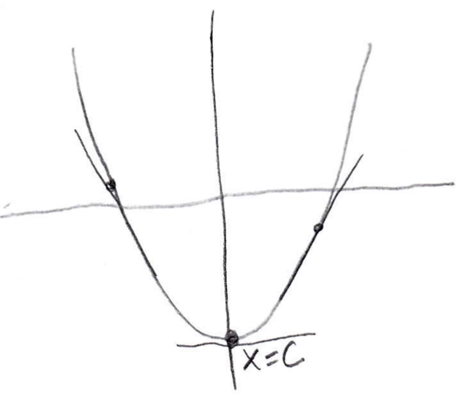
1) $f(c)$ is a <u>relative maximum</u> of f if there exists an interval (a,b) containing c such that $f(x) \le f(c)$ for all x in (a,b).
2) $f(c)$ is a <u>relative minimum</u> of f if there exists an interval (a,b) containing c such that $f(x) \ge f(c)$ for all x in (a,b).

If f has a relative extremum at $x=c$, then c is a critical number of f.

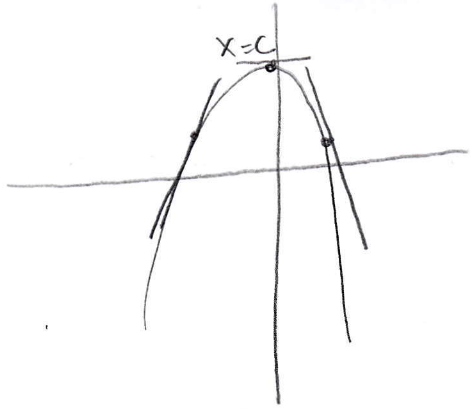
The first derivative test

Let f be continuous on the interval (a,b) in which c is the only critical number. If f is differentiable on the interval, then $f(c)$ can be classified by:

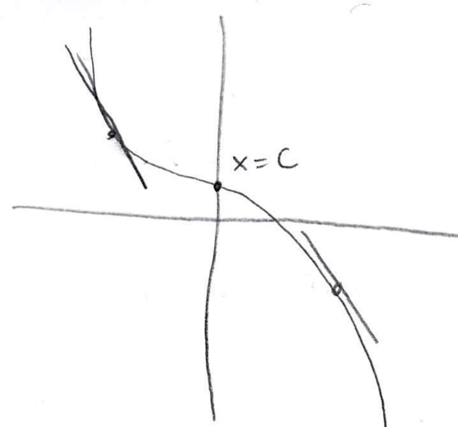
1) On (a,b), if $f'(x) < 0$ to the left of $x=c$ and $f'(x) > 0$ to the right of $x=c$, $f(c)$ is a relative minimum.
2) On (a,b), if $f'(x) > 0$ to the left of $x=c$ and $f'(x) < 0$ to the right, then $f(c)$ is a relative maximum.
3) On (a,b), if $f'(x)$ has the same sign to the left AND right of $x=c$, then $f(c)$ is NOT a relative extremum.



minimum



maximum



neither

<u>Ex</u> $f(x) = x^4 - x^3$

<u>Soln</u> $f'(x) = 4x^3 - 3x^2$

$$0 = x^2(4x-3)$$

$x = 0, \quad x = 3/4$ critical numbers.

Interval	Test	Sign	Conclusion
$(-\infty, 0)$	$x=-1$	$f'(-1) = -7 < 0$	decrease
$(0, 3/4)$	$x = \frac{1}{2}$	$f'(\frac{1}{2}) = -1/4 < 0$	decrease
$(3/4, \infty)$	$x = 1$	$f'(1) = 1 > 0$	increase

So $x = \frac{3}{4}$ yields a relative minimum.

Absolute Extrema

Def Let f be defined over an interval I containing c.

1) $f(c)$ is an absolute minimum of f on I if $f(c) \leq f(x)$ for all x in I.

2) $f(c)$ is an absolute maximum of f on I if $f(c) \geq f(x)$ for all x in I.

Extreme Value Theorem: If f is continuous on $[a,b]$, then f has both a maximum and a minimum on $[a,b]$.

Finding extrema on a closed interval:
For a continuous function f on the closed interval $[a,b]$,

1) Evaluate f at each critical number on $[a,b]$.
2) Evaluate f at the endpoints a and b.
3) The least of these values is the minimum.
   The greatest of these values is the maximum.

Ex Find the min and max of $f(x) = x^2 - 6x + 2$ on $[0, 5]$.

Soln $f'(x) = 2x - 6$

$0 = 2x - 6$

$x = 3$

$f(3) = -7$

Now, $f(0) = 2$ and $f(5) = -3$

So the minimum is -7 at $x = 3$

the maximum is 2 at $x = 0$.