Chapter 0 Review

Section 0.1

Rational or irrational?

1. $0.25 = \frac{1}{4}$ rational

3. $\frac{3\pi}{2} \rightarrow \pi$ is irrational

   So $a\pi$ is also irrational

5. $4.3\overline{451}$ is rational
   (there is a repeating pattern)

Note: In general, you can show a number is rational by writing it as a fraction

$$\frac{a}{b}$$

Some numbers π, $\sqrt{2}$, e are irrational and good to know about.

11. Does x satisfy the inequality? $5x - 12 > 0$

a) $x = 3$

Two approaches:

(1) Plug in x:

$$5(3) - 12 > 0$$
$$15 - 12 > 0$$
$$3 > 0 \quad \text{yes}$$

(2) Solve for x:

$$5x - 12 > 0$$
$$5x > 12$$
$$x > \frac{12}{5}$$
$$x > 2.4$$

b) $x = -3 \not> 2.4$ no

c) $x = \frac{5}{2} > 2.4$

$$2.5 > 2.4 \quad \text{yes}$$

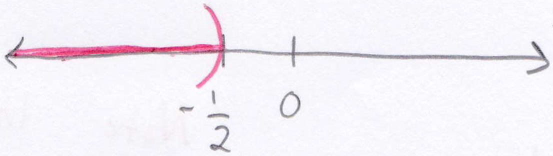
17. Solve and sketch the number line.

$4x+1 < 2x$

$2x+1 < 0$

$2x < -1$

$x < -\frac{1}{2}$



$(-\infty, -\frac{1}{2})$

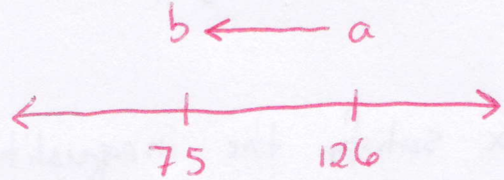
Section 0.2

Find the directed distance from a to b, b to a, and the distance

1. $a = 126 \quad b = 75$

a to b:

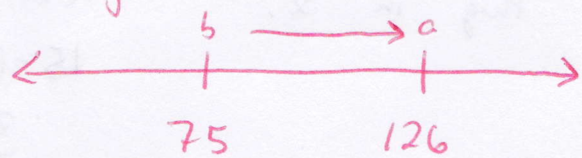
$b - a = 75 - 126 = -51$



This is in the negative direction so we expect the answer to be negative.

b to a:

$a - b = 126 - 75 = 51$



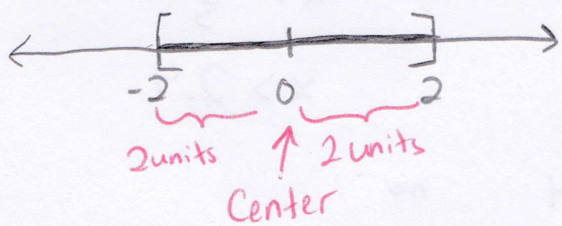
pos. direction – answer should be pos.

distance:

$|a-b| = |b-a| = 51$

Use absolute values to describe the interval.

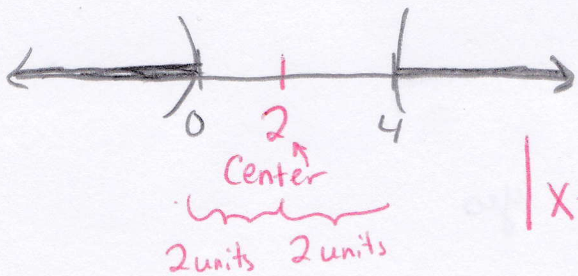
7. $[-2, 2]$



$|x| \leq 2$

$|x - \text{Center}| \leq \left(\begin{array}{c}\text{\# of} \\ \text{units} \\ \text{to end}\end{array}\right)$

13. $(-\infty, 0) \cup (4, \infty)$



$|x-2| > 2$

$|x - \text{Center}| > \left(\begin{array}{c}\text{\# of} \\ \text{units to} \\ \text{end of} \\ \text{inside interval}\end{array}\right)$

33. Solve and sketch $\left|\frac{3x-a}{4}\right| < 2b, \quad b>0$

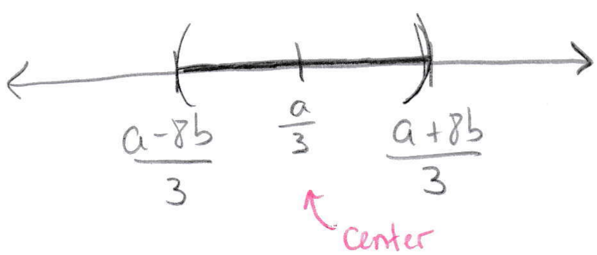
$$\left|\frac{3x-a}{4}\right| < 2b$$

$$-2b < \frac{3x-a}{4} < 2b \qquad \text{Convert from absolute value}$$

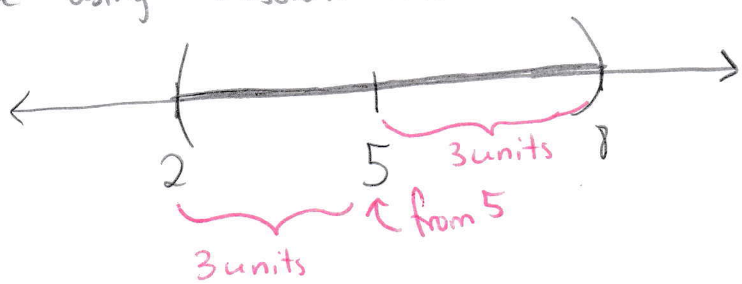
$$-8b < 3x-a < 8b \qquad \text{multiply by } 4$$

$$a-8b < 3x < a+8b \qquad \text{add } a$$

$$\frac{a-8b}{3} < x < \frac{a+8b}{3} \qquad \text{divide by } 3$$



15. Write using absolute values: all numbers less than 3 units from 5.



Check: 1 is 4 units from 5
(not in interval)
3 is 2 from 5
(in interval)
9 is 4 from 5
(not in interval.)

$|x - \text{center}| <$ units from end of interval

$$|x-5| < 3$$

35. Find the midpoint of $[8, 24]$

$$\text{Midpoint} = \frac{8+24}{2} = 16$$

Section 0.3

21. $6y^{-2}(2y^{4})^{-3}$

$= 6y^{-2}\, 2^{-3}\, y^{4(-3)}$

$= 6 \times 2^{-3}\, y^{-2}\, y^{-12}$

$= \frac{6}{2^3}\, y^{-2-12}$

$= \frac{6}{8}\, y^{-14}$

$= \frac{3}{4y^{14}}$

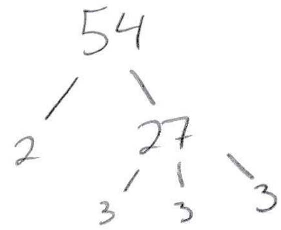
33. Factor out as much as possible:

$\sqrt[3]{54x^5}$

$= \sqrt[3]{27 \cdot 2 x^2 x^3}$

$= \sqrt[3]{27x^3}\sqrt[3]{2x^2}$

$= 3x\sqrt[3]{2x^2}$



$\star\ 27 = 3^3$

$\Rightarrow 3 = \sqrt[3]{27}$

Section 0.4

37. Find the real zeros of (x^2-9)

$0 = x^2 - 9$

$= (x+3)(x-3)$

$x + 3 = 0$ and $x - 3 = 0$

$x = -3$ and $x = 3$ are the real zeros.

39. Find the real zeros of (x^2-3)

$0 = (x^2 - 3)$

$= (x + \sqrt{3})(x - \sqrt{3})$

$x + \sqrt{3} = 0$ and $x - \sqrt{3} = 0$

$x = -\sqrt{3}$ and $x = \sqrt{3}$ are the real zeros.

41. $(x-3)^2 - 9 = 0$ Find the real zeros.

$$(x^2 - 6x + 9) - 9 = 0$$

$$x^2 - 6x = 0$$

$$x(x-6) = 0$$

$x = 0$ and $x - 6 = 0$

$x = 0$ and $x = 6$ are the real zeros.

Section 0.5

7. $\dfrac{5}{x-3} + \dfrac{3}{3-x}$ Simplify

$$= \frac{5}{x-3} + \frac{3}{-(x-3)}$$

$$= \frac{5}{x-3} - \frac{3}{x-3}$$

$$= \frac{2}{x-3}$$

Note: If you started with

$$\frac{5(3-x)}{(x-3)(3-x)} + \frac{3(x-3)}{(3-x)(x-3)}$$

you would get the same answer, but you'd work a lot harder to get there!

13. Add. $\dfrac{-2}{x} + \dfrac{1}{x^2+2}$

$$-\frac{2}{x}\left(\frac{x^2+2}{x^2+2}\right) + \frac{1}{x^2+2}\left(\frac{x}{x}\right)$$

$$= \frac{-2x^2-4}{x(x^2+2)} + \frac{x}{x(x^2+2)}$$

$$= \frac{-2x^2+x-4}{x(x^2+2)}$$

44. Simplify and rationalize.

$$\frac{\dfrac{\sqrt{x^2+1}}{x^2} - \dfrac{1}{x\sqrt{x^2+1}}}{x^2+1}$$

I am going to work on one piece at a time:

$$\frac{\sqrt{x^2+1}}{x^2} - \frac{1}{x\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}}{x^2}\left(\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}\right) - \frac{1}{x\sqrt{x^2+1}}\left(\frac{x}{x}\right)$$

$$= \frac{x^2+1}{x^2\sqrt{x^2+1}} - \frac{x}{x\sqrt{x^2+1}}$$

$$= \frac{x^2-x+1}{x^2\sqrt{x^2+1}}$$

So

$$\frac{\dfrac{\sqrt{x^2+1}}{x^2} - \dfrac{1}{x\sqrt{x^2+1}}}{x^2+1} = \frac{\dfrac{x^2-x+1}{x^2\sqrt{x^2+1}}}{\dfrac{x^2+1}{1}}$$

$$= \frac{x^2-x+1}{x^2\sqrt{x^2+1}}\left(\frac{1}{x^2+1}\right)$$

$$= \frac{x^2-x+1}{x^2(x^2+1)^{1/2}(x^2+1)^1}$$

$$= \frac{x^2-x+1}{x^2(x^2+1)^{3/2}}$$

Note: with more complex algebraic expressions, there may be multiple correct ways to simplify.