Practice Final

1) $B = 1.71 + 0.29t$, $\quad 0 \le t \le 15$

a) slope $m = 0.29$ $\quad$ intercept $(0, 1.71)$

$\cancel{\left(-\frac{1.71}{0.29}, 0\right)}$ outside range of t

b) $t = 6$

$B = 1.71 + 0.29(6)$

$= 3.45$

2) $h(x) = 2x^2 + x$. Then find the slope of the tangent line at $(1,3)$

$$\lim_{\Delta x \to 0} \frac{h(x+\Delta x) - h(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\left[2(x+\Delta x)^2 + (x+\Delta x)\right] - \left[2x^2 + x\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{2x^2} + 4x\Delta x + (\Delta x)^2 + \cancel{x} + \Delta x - \cancel{2x^2} - \cancel{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4x\cancel{\Delta x} + (\Delta x)^{\cancel{2}} + \cancel{\Delta x}\,1}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} 4x + \Delta x + 1$$

now our function is a polynomial so we can plug in $\Delta x = 0$.

$= 4x + 1$

3) $f(x) = \dfrac{(x+2)^2}{x-2}$ $\qquad \frac{d}{dx}(x+2)^2 = 2(x+2) = 2x+4$ chain rule

$$f'(x) = \frac{(x-2)(2x+4) - (x+2)^2(1)}{(x-2)^2}$$ quotient rule

$$= \frac{2x^2 + 4\cancel{x} - \cancel{4x} - 8 - (x^2 + 4x + 4)}{(x-2)^2}$$

$$\boxed{= \frac{x^2 - 4x - 12}{(x-2)^2}} = \frac{(x-6)(x+2)}{(x-2)^2}$$

$\frac{d}{dx}[(x-2)^2]$ & chain rule

$2(x-2)$

$2x-4$

$$f''(x) = \frac{(x-2)^2(2x-4) - (x^2-4x-12)(2x-4)}{(x-2)^4}$$ quotient rule

$$= \frac{(2x-4)\left[\cancel{x^2} - \cancel{4x} + 4 - \cancel{x^2} + \cancel{4x} + 12\right]}{(x-2)^4}$$

$$= \frac{2\cancel{(x-2)}(16)}{(x-2)^{\cancel{4}3}}$$

$$= \frac{32}{(x-2)^3}$$

$$= 32(x-2)^{-3}$$

$$f''(x) = 32(-3)(x-2)^{-4}(1)$$

$$= -96(x-2)^{-4}$$

$$= \frac{-96}{(x-2)^4}$$

4) $g(x) = \dfrac{(x^3+4)^2}{3} = \frac{1}{3}(x^3+4)^2$

$g'(x) = \frac{1}{\cancel{3}}\left[2(x^3+4)(\cancel{3}x^2)\right]$

$= 2x^5 + 8x^2$

$g''(x) = 10x^4 + 16x = 0$ $\qquad x = 0$

$10x^3 + 16 = 0$ $\qquad x \neq 0$

$x^3 + 1.6 = 0$

$x = \sqrt[3]{-1.6}$

$x \approx -1.17$

Interval	Test	Sign of g''	Conclusion
* $(-\infty, -1.17)$	$x=-2$	$+128$	upward
* $(-1.17, 0)$	$x=-1$	-6	downward
* $(0, \infty)$	$x=1$	$+26$	upward

5) $g(x) = 2x^3 - 5x^2 - 4x + 11$ $\qquad$ First derivative test

$g'(x) = 6x^2 - 10x - 4 = 0$

$g''(x) = 12x - 10$

$6x^2 - 10x - 4 = 0$

$x^2 - \frac{5}{3}x - \frac{2}{3} = 0$

$x = 2 \qquad x = -\frac{1}{3}$

Interval	Test	Sign of f'	
$(-\infty, -\frac{1}{3})$	$x=-1$	$+12$	Increase
$(-\frac{1}{3}, 2)$	$x=0$	-4	Decrease
$(2, \infty)$	$x=3$	$+20$	Increasing

Relative max at $x = -\frac{1}{3}$

Relative min at $x = 2$

6) $f(x) = \dfrac{(2e^x)^2}{e^5} + 7x^2$

a) $f(x) = \dfrac{4e^{2x}}{e^5} + 7x^2$

$= 4e^{2x-5} + 7x^2$

b) $f'(x) = 4\frac{d}{dx}\left[e^{2x-5}\right] + 14x$ $\qquad u = 2x-5$, $u' = 2$

$= 4(e^u u') + 14x$

$= 4(2e^{2x-5}) + 14x$

$= 8e^{2x-5} + 14x$

7) $g(x) = \ell\left(\dfrac{4x+2}{x^2}\right)$

a) $= \ell(4x+2) - \ell(x^2)$

$= \ell(4x+2) - 2\ell(x)$

$= \ell[2(x+1)] - 2\ell(x)$

$= \ell(2) + \ell(x+1) - 2\ell(x)$

b) $g'(x) = 0 + \frac{1}{x+1}(1) - 2\left(\frac{1}{x}\right)$

$= \frac{1}{x+1} - \frac{2}{x}$ $\qquad$ Simplify into a single fraction.

8) $\int (2x^3 - x^2 + 7x^1 + 2)\,dx = 2\int x^3 dx - \int x^2 dx + 7\int x\,dx + \int 2\,dx$

$= 2\left[\frac{x^4}{4}\right] - \frac{x^3}{3} + 7\frac{x^2}{2} + 2x + C$

$= \frac{1}{2}x^4 - \frac{1}{3}x^3 + \frac{7}{2}x^2 + 2x + C$

Confirm by differentiating:

$\frac{d}{dx} \to \frac{4}{2}x^3 - \frac{3}{3}x^2 + \frac{7}{2}(2)x + 2$

$= 2x^3 - x^2 + 7x + 2$ ✓

9) $\displaystyle\int \frac{3x^2 - 2}{\sqrt{2x^3 - 4x + 3}}\,dx = \int (3x^2-2)(2x^3-4x+3)^{-1/2}\,dx$

let $u = 2x^3 - 4x + 3$ $\qquad n = -\frac{1}{2}$

$\frac{du}{dx} = 6x^2 - 4$

$du = (6x^2-4)dx$

$du = 2(3x^2-2)dx$

$\frac{1}{2}du = (3x^2-2)dx$

$= \int u^{-1/2}\left(\frac{1}{2}\right)du$

$= \frac{1}{2}\int u^{-1/2}\,du$

$= \cancel{\frac{1}{2}}\left(\frac{u^{1/2}}{\cancel{1/2}}\right) + C$

$= u^{1/2} + C = (2x^3 - 4x + 3)^{1/2} + C$

$\boxed{EC}$ $\displaystyle\int (3x^2+1)\,e^{\frac{x^3+x}{2}}\,dx$

$u = \frac{1}{2}(x^3+x)$

$\frac{du}{dx} = \frac{1}{2}(3x^2+1)$

$du = \frac{1}{2}(3x^2+1)dx$

$2du = (3x^2+1)dx$

$= \int 2e^u\,du$

$= 2e^u + C$

$= 2e^{\frac{x^3+x}{2}} + C$