Chapter 2 Review (p 182)

1. $f(x) = x^2 + 1$ $(2, 5)$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{[(x+\Delta x)^2 + 1] - (x^2+1)}{\Delta x}$$

$$= \frac{[\cancel{x^2} + 2x\Delta x + (\Delta x)^2 \cancel{+1}] - (\cancel{x^2} \cancel{+1})}{\Delta x}$$

$$= \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= 2x + (\Delta x) \qquad \Delta x \neq 0$$

$$\lim_{\Delta x \to 0}\left[\frac{f(x+\Delta x) - f(x)}{\Delta x}\right] = \lim_{\Delta x \to 0}(2x + \Delta x) = 2x$$

At $x = 2$, $f'(x) = 2x = 2 \times 2 = 4$

2) $f(x) = \sqrt{x} - 2$ $\quad\quad (4,0)$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\left(\sqrt{x+\Delta x} \cancel{-2}\right) - \left(\sqrt{x} \cancel{-2}\right)}{\Delta x} = \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$$

$$\frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}\left(\frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}\right) = \frac{(\cancel{x} + \Delta x) - \cancel{x}}{\Delta x\left[\sqrt{x+\Delta x} + \sqrt{x}\right]}$$

$$= \frac{\cancel{\Delta x}}{\cancel{\Delta x}\left(\sqrt{x+\Delta x} + \sqrt{x}\right)} = \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$\lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

At $x=4$, $f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

3. $f(t) = t^3 + 2t$

$f'(t) = 3t^2 + 2$

5. $f(x) = x^{3/2}$

$f'(x) = \frac{3}{2}x^{3/2 - 1}$

$= \frac{3}{2}x^{1/2}$

6) $f(x) = (x+3)(x-3)$

A) $x^2 - 9 = f(x)$

$f'(x) = 2x$

B) Product Rule

$f'(x) = (x+3)(1) + (x-3)(1)$

$= x + 3 + x - 3$

$= 2x$

7) $f(x) = -3x^{-3}$

$f'(x) = -3(-3)x^{-4}$

$= 9x^{-4}$

8) $f(x) = \sqrt{x}(5+x) = x^{1/2}(5+x)$

$f'(x) = x^{\frac{1}{2}}(1) + (5+x)\left[\frac{1}{2}x^{-1/2}\right]$

$= x^{\frac{1}{2}} + \frac{5}{2}x^{-1/2} + \frac{1}{2}x^{1/2}$

$= \frac{3}{2}x^{\frac{1}{2}} + \frac{5}{2}x^{-1/2}$

$= \left(\frac{\sqrt{x}}{\sqrt{x}}\right)\frac{3\sqrt{x}}{2} + \frac{5}{2\sqrt{x}}$

$= \frac{3x}{2\sqrt{x}} + \frac{5}{2\sqrt{x}}$

$= \frac{3x+5}{2\sqrt{x}}$

9) $f(x) = (\underbrace{3x^2+4}_{u})^2$

$y = u^2$

$\frac{dy}{du} = 2u \qquad \frac{du}{dx} = 6x$

$f'(x) = \frac{dy}{du}\frac{du}{dx} = 2u(6x)$

$= 2(3x^2+4)(6x)$

$= 12x(3x^2+4)$

10) $f(x) = \sqrt{1-2x} = (\underbrace{1-2x}_{u})^{\frac{1}{2}}$

$y = u^{1/2}$

$f'(x) = \left(\frac{1}{2}u^{-1/2}\right)(-2)$

$= -u^{-1/2} = -(1-2x)^{-1/2}$

$= -\frac{1}{\sqrt{1-2x}}$

11) $f(x) = \frac{(5x-1)^3}{x}$

$u = 5x-1$

$y = u^3$

$f'(x) = \frac{x\frac{d}{dx}\left[(5x-1)^3\right] - (5x-1)^3\frac{d}{dx}[x]}{x^2}$

$= \frac{x\left[(3u^2)(5)\right] - (5x-1)^3(1)}{x^2}$

$= \frac{x\left[15(5x-1)^2\right] - (5x-1)^3}{x^2} = \frac{15x(5x-1)^2 - (5x-1)^3}{x^2}$

D) $f(x) = x - \frac{1}{x}$ at $(1,0)$

$$= x - x^{-1}$$

$$f'(x) = 1 - (-1x^{-2})$$

$$= 1 + x^{-2}$$

$$= 1 + \frac{1}{x^2}$$

At $x=1$ $\quad f'(1) = 1 + \frac{1}{1^2} = 2$

$$y - y_0 = m(x - x_0)$$

$$y - 0 = 2(x-1)$$

$$y = 2x - 2$$

14) Find the third derivative

$$f(x) = 2x^2 + 3x + 1$$

$$f'(x) = 4x + 3$$

$$f''(x) = 4$$

$$f^{(3)}(x) = 0$$

15) $f(x) = \sqrt{3-x} = (3-x)^{1/2}$

$$f'(x) = \frac{1}{2}(3-x)^{-1/2}(-1) = -\frac{1}{2}(3-x)^{-1/2}$$

$$f''(x) = -\frac{1}{2}\left(-\frac{1}{2}\right)(3-x)^{-3/2}(-1)$$

$$= -\frac{1}{4}(3-x)^{-3/2}$$

$$f^{(3)}(x) = -\frac{1}{4}\left(-\frac{3}{2}\right)(3-x)^{-5/2}(-1)$$

$$= -\frac{3}{8}(3-x)^{-5/2}$$

16) $f(x) = \frac{2x+1}{2x-1}$

$$f'(x) = \frac{(2x-1)(2) - (2x+1)(2)}{(2x-1)^2} = \frac{\cancel{4x} - 2 - (\cancel{4x} + 2)}{(2x-1)^2} = \frac{-4}{(2x-1)^2}$$

$$= -4(2x-1)^{-2}$$

$$f''(x) = -4\left[-2(2x-1)^{-3}(2)\right]$$

$$= 16(2x-1)^{-3}$$

$$f'''(x) = 16\left[-3(2x-1)^{-4}(2)\right]$$

$$= -96(2x-1)^{-4}$$

$$= -\frac{96}{(2x-1)^4}$$