3   2   47   118   204   97   68   86   62   57   98   99

3.1) Find the mean, median, and mode.

a) Mean

$$\frac{3+2+47+\ldots+98+99}{12} = 78.42 = \bar{x}$$
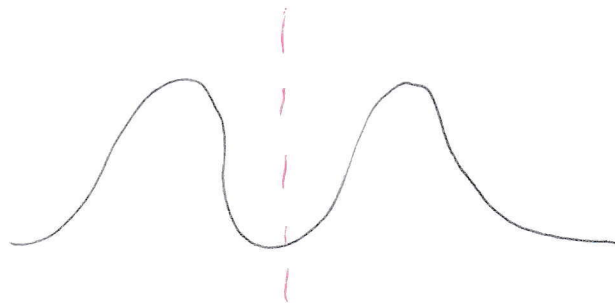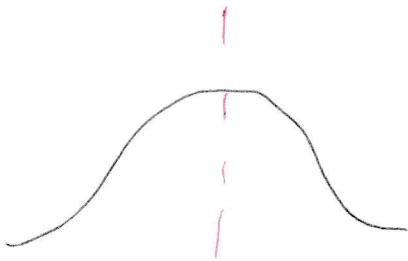
b) median

Sort data:

2   3   47   57   62   68 | 86   97   98   99   118   204

$$\frac{n+1}{2} = \frac{13}{2} = 6.5$$

$$\text{Median:} \quad \frac{68+86}{2} = 77$$

c) Mode: No number appears more than once → there is no mode!

Mean ≈ median → roughly symmetric → use mean.
         as   measure of center.

Find the range and sample standard deviation.

Range: Max − Min = 204 − 2 = 202

Standard deviation

Formula #1

$$S = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{12} (X_i - 78.42)^2}{11}}$$

$X - \bar{X}$	$(X - \bar{X})^2$
−75.42	5687.67
−76.42	5839.51
−31.42	987.01
39.58	1566.84
125.58	15771.17
⋮	⋮

total: 31898.92

$$= \sqrt{\frac{31898.92}{11}}$$

$$= \sqrt{2899.90}$$

$$= 53.85$$

Formula #2

$$S = \sqrt{\frac{\sum(X_i^2) - (\sum X_i)^2/n}{n-1}}$$

X	X^2
3	9
2	4
47	2209
118	13924
204	41616
⋮	⋮
941	105689

$$= \sqrt{\frac{105689 - (941)^2/12}{11}}$$

$$= 53.85$$

3.3 Apply Chebyshev's rule with $k=2$ and $k=3$

a) <u>At least 75% of the observations fall within two sd of</u>
the mean.

75% of 12 → $0.75(12) = 9$

$\bar{x} \pm 2s$ → $78.42 \pm 2(53.85)$
-29.28 to 186.12

At least 9 of the observations fall within -29.28 to 186.12
tornadoes.

In fact $^{11}/_{12}$ obs. fall in this range   (91.6%)

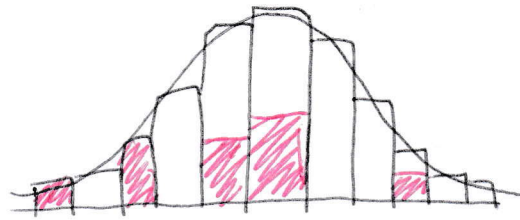b) <u>At least 89% of the observations fall w/in three sd of</u>
the mean.

89% of 12 → $0.89 \times 12 = 10.68$ → 11

$\bar{x} \pm 3s$ → $78.42 \pm 3(53.85)$
-83.13 to 239.97

In fact $^{12}/_{12}$ obs fall in this range   (100%)

At least 11 of the obs. fall w/in ~~the~~ -83.13 to
239.97.

Empirical rule:

$\simeq 68\%$ of the obs. fall within one sd of the mean.

$\bar{x} \pm s \longrightarrow \quad 78.45 \pm 53.85$

24.57 to 132.27

9/12 fall in this range (75%)

8/12 = 0.67

$\simeq 95\%$ of the obs. fall w/in two sd of the mean

$\bar{x} \pm 2s \longrightarrow \quad -29.28$ to 186.12

11/12 fall in this range (91.6%)

$\simeq 99.7\%$ fall (within 3 sd of the mean)

$\bar{x} \pm 3s \longrightarrow \quad -83.13$ to 239.97

12/12 (100%) fall in this range.