

Another formula for sample standard deviation:

$$S = \sqrt{\frac{\sum x_i^2 - (\sum x_i)^2/n}{n-1}}$$

Where n is the sample size.

Ex For Team 2

x	x^2
67	4489
72	5184
76	5776
76	5776
84	7056
375	28,281

$$\sum x_i^2 = 28281$$

$$(\sum x_i)^2 = 375^2$$

$$\begin{aligned} \text{So, } S &= \sqrt{\frac{28281 - (375)^2/5}{5-1}} = \sqrt{\frac{28281 - 28125}{4}} \\ &= \sqrt{\frac{156}{4}} = 6.2 \end{aligned}$$

The range of a data set is the difference b/w the largest and smallest observations.

$$\text{Range} = \text{Max} - \text{Min}$$

For our bball teams,

$$\text{Team 1: Range} = 78 - 72 = 6 \text{ inches}$$

$$\text{Team 2: Range} = 84 - 67 = 17 \text{ inches}$$

Easy to calculate, but leaves out a lot of information!

Sample Standard Deviation

Takes into account all observations.

* We can think of variability as how far the observations are from the mean. For an observation x_i ,

$x_i - \bar{x}$ is the deviation.

For Team 1,

$$\bar{x} = \frac{\sum x_i}{n} = \frac{72 + 73 + 76 + 76 + 78}{5} = \frac{375}{5} = 75 \text{ in}$$

The deviation for $x_1 = 72$ is

$$72 - 75 = -3$$

X	deviation
72	-3
73	-2
76	1
76	1
78	3

What if we tried average deviation?

Sum of deviations is always zero!

$$-3 - 2 + 1 + 1 + 3 = 0 !$$

Instead, we square the deviations:

$(x_i - \bar{x})^2$ and calculate the

Sum of squared deviations

$$\sum (x_i - \bar{x})^2$$

This gives a measure of total deviation from the mean.

x	deviation $x - \bar{x}$	Squared deviation $(x - \bar{x})^2$
72	-3	9
73	-2	4
76	1	1
76	1	1
78	3	9

$$\sum (x_i - \bar{x})^2 = 24 \text{ inches}^2$$

To get variance, divide by $n-1$

$$\frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{24}{4} = 6$$

Def The Sample variance of a variable X is denoted S_x^2 (or s^2).

$$S_x^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

Def The Sample standard deviation is denoted S_x (or s)

$$S_x = \sqrt{S_x^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

For Team 1,

$$S_x = \sqrt{S_x^2} = \sqrt{6} = 2.4.$$

On average, the heights of the players on Team 1 vary from the mean height by about 2.4 inches.

Procedure:

- 1) Calculate Sample mean $\bar{x}$.
- 2) Construct a table to calculate $(x_i - \bar{x})$, $(x_i - \bar{x})^2$
and the sum $\Sigma(x_i - \bar{x})^2$
- 3) Divide by $n-1$. For S , take the square root.

Ex Find the sample standard deviation for Team 2: 67, 72, 76, 76, 84

Soln 1) Mean: $\frac{67+72+76+76+84}{5} = \frac{375}{5} = 75 \text{ in}$

2) Table

x	$x - \bar{x}$	$(x - \bar{x})^2$
67	-8	64
72	-3	9
76	1	1
76	1	1
84	9	81

Total: 156 in²

3) $s^2 = \frac{\sum (x - \bar{x})^2}{n-1} = \frac{156}{5-1} = 39$

$$s = \sqrt{s^2} = \sqrt{39} \approx 6.2 \text{ in}$$