Chebyshev's Rule: For any quantitative data set and any real number $k \geq 1$, at least $1 - \frac{1}{k^2}$ of the observations lie within k standard deviations of the mean. That is, $1 - \frac{1}{k^2}$ of observations lie between $\bar{x} - ks$ and $\bar{x} + ks$.

Special case:

$k = 2 \rightarrow 1 - \frac{1}{2^2} = 1 - \frac{1}{4} = 0.75$

75% of observations lie within 2 standard devs.

$k = 3 \rightarrow 1 - \frac{1}{3^2} = 1 - \frac{1}{9} \approx 0.89$

89% of obs. lie within 3 standard deviations of the mean.

a. Apply Chebyshev's rule with $k = 2$ to make pertinent statements about the sample.

$\bar{x} = 18.8$ $s = 1.12$ $n = 140$

$18.8 \pm 2(1.12)$

18.8 ± 2.24

16.56 to 21.04

$140 \times 0.75 = 105$

At least 105 men have forearm length between 16.56 and 21.04 in.

b. Repeat with $k = 3$.

$18.8 \pm 3(1.12)$

18.8 ± 3.36

15.44 to 22.16

$140 \times 0.89 = 124.6 \rightarrow 125$

"at least" ⇒ round up!

At least 125 men have forearm length between 15.44 and 22.16 in.

Empirical rule: For quantitative data with a roughly bell-shaped distribution,

(1) Approximately 68% of the data fall within one standard deviation of the mean:

$$68\% \text{ within } \bar{x} \pm s$$

(2) Approximately 95% of the observations fall within two standard deviations of the mean, $\bar{x} \pm 2s$

(3) Approximately 99.7% of the observations fall within three standard deviations of the mean,

$$\bar{x} \pm 3s.$$

Ex Based on the table data,

$\bar{x} = 206.45$ ppm and $s = 66.42$ ppm.

a) Yes - roughly bell-shaped.

b) 68% within one std. dev.
95% within two std dev.
99.7% within three std dev.

Based on the table,
43 are within one std. dev.
$43/60 = 71.7\%$
$57/60 = 95\%$ within two
$60/60 = 100\%$ within three

c) $\bar{x} = 206.45$

$\bar{x} \pm s = 206.45 \pm 66.42$ is 140.03 to 272.87

$206.45 \pm 2 \times 66.42$ is 73.61 to 339.29

$206.45 \pm 3 \times 66.42$ is 7.19 to 405.71