The Multiplication Rule

$$P(B|A) = \frac{P(A \& B)}{P(A)}$$

If A and B are any two events, then

$$P(A \& B) = P(A)\,P(B|A) = P(B)\,P(A|B)$$

Ex 113th Congress 18.7% are senators, 53% of Senators are Democrats. Find Prob of Democratic senator.

D = event Democrat

S = event Senator

$P(S) = 0.187$

D & S

$P(D|S) = 0.53$

$P(D \& S) = P(S)\,P(D|S) = 0.187 \times 0.53 = 0.09$

Pet	Frequency
Dogs	17
Cats	23
	40

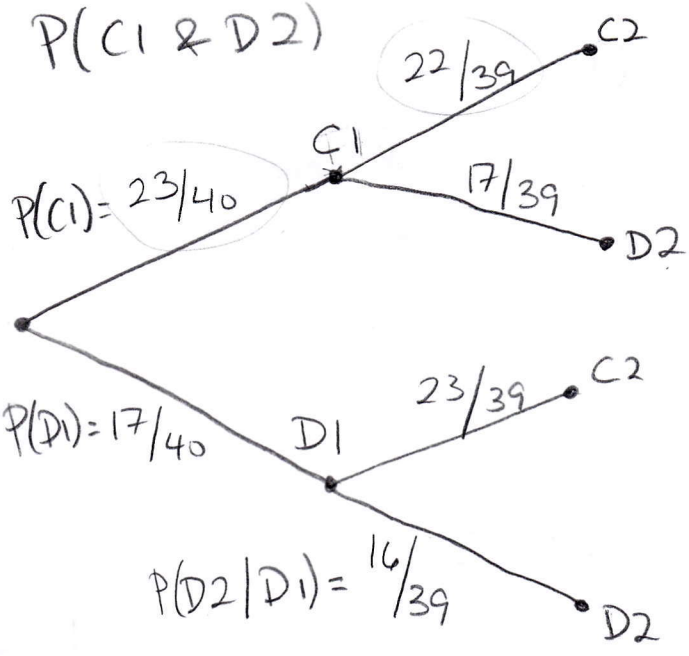
Draw two at random

Determine probability draw cat and then dog.

C1 = event first draw cat

D2 = event second draw dog

$P(C1 \& D2)$



$P(C1 \& C2) = \frac{23}{40} \cdot \frac{22}{39} = 0.324$

$P(C1 \& D2) = \frac{23}{40} \cdot \frac{17}{39} = 0.251$

$2 \times 2 = 4$

Independence

Event B is <u>independent of</u> event A if $P(B|A) = P(B)$.

Ex Playing cards

F = event face card

K = event King

H = event heart.

$P(K) = \frac{4}{52} = \frac{1}{13} = 0.077$

a) Is K independent of F? Find $P(K|F)$

$P(K|F) = \frac{4}{12} = \frac{1}{3} = 0.333 \neq P(K)$ dependent events

K is NOT independent of F.

b) $P(K|H) = \frac{1}{13} = 0.077 = P(K)$ independent events

$P(A \& B) = P(A)\ P(B|A)$

If A and B are independent, $P(B|A) = B$, $P(A|B) = A$

$P(A \& B) = P(A)P(B)$.

if $P(A \& B) = P(A)\ P(B)$, then A and B are independent events.

if A, B, C, ... are independent, then

$P(A \& B \& C \& \ldots) = P(A)\ P(B)\ P(C) \ldots$

Mutually Exclusive versus Independent Events

If two events are mutually exclusive, are they independent?

$P(A \& B) = 0$ mutually exclusive.