Counting Rules

$\frac{f}{N}$

Basic Counting rule:

r actions to be performed in a definite order

If there are m_i possibilities for the i^{th} action

Then the total possibilities

$$m_1 m_2 m_3 \cdots m_r$$

Ex License Plates 3 letters then 3 numbers.

How many are possible?

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17,576,000$$

How many with no repeated characters?

$$26\ 25\ 24\ 10\ 9\ 8 = 11,232,000$$

Factorials (k is a positive integer)

$$k! = k(k-1)(k-2)\cdots(2)(1)$$

Define $0! = 1$

Ex $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Permutations

r objects from a collection of m objects

order matters!

$$_mP_r = \frac{m!}{(m-r)!}$$

Ex 10 books to arrange on a shelf

10 books from a collection of 10 books.

$$_{10}P_{10} = \frac{10!}{(10-10)!} = \frac{10!}{0!} = \frac{10!}{1} = 3,628,800$$

$$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 10!$$

Combinations

r objects from a collection of m objects

order does not matter!

$$_mC_r = \frac{m!}{r!(m-r)!}$$

Want to get a random sample of 5 students from a class of 35 students. How many possible samples are there?

$$_{35}C_5 = \frac{35!}{5!(35-5)!} = \frac{35!}{5!(30)!} = \frac{35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 \cdot \cancel{30!}}{5! \cdot \cancel{30!}}$$

$$= 324,632$$

Number of possible samples from a pop of size N $\quad _NC_n$