Mean of a Discrete Random Variable

$$\mu = \frac{\Sigma X_i}{N}$$

The mean of a discrete random variable X, μ_X or μ, is

$$\mu = \Sigma x \, P(X = x)$$

Also called expected value or expectation.

Ex Six-Sided die $P(X=x) = \frac{1}{6}$ $x = 1, 2, 3, 4, 5, 6$

$$\mu = 1\left(\tfrac{1}{6}\right) + 2\left(\tfrac{1}{6}\right) + 3\left(\tfrac{1}{6}\right) + 4\left(\tfrac{1}{6}\right) + 5\left(\tfrac{1}{6}\right) + 6\left(\tfrac{1}{6}\right)$$

$$= \frac{1+2+3+4+5+6}{6}$$

$$= 3.5$$

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$$= \frac{1+2+3+4+5+6}{6}$$

Note We may also use expected value or expectation in place of "mean".

Ages = X
19
20
20
19
21
27
20
21

$$\mu = \frac{\Sigma x_i}{N} = \frac{19+20+20+19+21+27+20+21}{8}$$

$$= \frac{19+19+20+20+20+21+21+27}{8}$$

$$= \frac{2\times 19 + 3\times 20 + 2\times 21 + 27}{8}$$

$$= 19\left(\tfrac{2}{8}\right) + 20\left(\tfrac{3}{8}\right) + 21\left(\tfrac{2}{8}\right) + 27\left(\tfrac{1}{8}\right) \quad \star$$

$$= 19\,P(X=19) + 20\,P(X=20) + 21\,P(X=21) + 27\,P(X=27)$$

$$= \Sigma x\,P(X=x)$$

Age X	$P(X=x)$
19	$2/8 = 0.250$
20	$3/8 = 0.375$
21	$2/8 = 0.250$
27	$1/8 = 0.125$

Interpretation

$\mu_x = 2.3$

In a large number of independent observations of a random variable X, the average value of the observations will approximately equal the mean μ of X.

Further, the larger the number of observations, the closer the average will tend to be to μ.

This principle is referred to as the <u>law of averages</u> or <u>the law of large numbers</u>.

Standard Deviation of a Discrete Random Variable

denoted σ_X or σ

$$\sigma = \sqrt{\Sigma (x_i - \mu)^2 P(X = x_i)}$$

OR

$$\sigma = \sqrt{\Sigma x_i^2 P(X = x) - \mu^2}$$

As before

σ^2 is the variance of X.

x	$P(X=x)$	$x\,P(X=x)$
0	0.029	0.000
1	0.049	0.049
2	0.078	0.156
3	0.155	0.465
4	0.212	0.848
5	0.262	1.310
6	0.215	1.290
		$\mu = 4.118$

X = # of tellers busy with customers at 1pm

$$\sigma = \sqrt{\Sigma x^2 P(X=x) - \mu^2}$$

$$= \sqrt{19.438 - (4.118)^2}$$

$$= 1.6$$

x^2	$x^2 P(X=x)$
0	0
1	0.049
4	0.312
9	1.395
16	3.392
25	6.550
36	7.740
	19.438

On average, the number of busy tellers varies around the mean of 4.118 by 1.6.