4.1 Sample Spaces and Events

Dr. Lauren Perry

Goals

1. Understand terminology related to probability.
2. Use probability notation.
3. Find and interpret probabilities for equally likely events.

Probability

Probability is the science of uncertainty.

- When we run an experiment, we are unsure of what the outcome will be.
- Because of this uncertainty, we say an experiment is a **random process**.

Probability

The probability of an event is the proportion of times it would occur if the experiment were run infinitely many times.

For a collection of *equally likely events*, this looks like:

$$\text{probability of event} = \frac{\text{number of ways event can occur}}{\text{number of possible outcomes}}$$

An **event** is some specified possible outcome (or collection of outcomes) we are interested in observing.

Example

You want to roll a 6 on a six-sided die.

- There are six possible outcomes $\{1, 2, 3, 4, 5, 6\}$.
- We assume that each die face is equally likely to appear on a single roll of the die.
  - The die is *fair*.
- The probability of rolling a 6 is

$$\frac{\text{number of ways to roll a 6}}{\text{number of possible rolls}} = \frac{1}{6}$$

Example

We can extend this to a collection of events.

- The probability of rolling a 5 or a 6:

$$\frac{\text{number of ways to roll a 5 or 6}}{\text{number of possible rolls}} = \frac{2}{6}$$

Sample Spaces

The collection of all possible outcomes is called a **sample space**, denoted S.

- For the six-sided die, $S = \{1, 2, 3, 4, 5, 6\}$.

Probability Notation

To simplify our writing, we use **probability notation**:

- Events are assigned capital letters.
- $P(A)$ denotes the probability of event A.
- Sometimes we will also shorten simple events to just a number.
  - For example, $P(1)$ might represent "the probability of rolling a 1".

We can estimate probabilities from a sample using a frequency distribution.

Example: Consider the frequency distribution.

Class	Frequency
freshman	12
sophomore	10
junior	3
senior	5

Class	Frequency
freshman	12
sophomore	10
junior	3
senior	5

If a student is selected at random, the probability of selecting a sophomore is

$$\frac{10 \text{ sophomores}}{30 \text{ students}} \approx 0.3333$$

Class	Frequency
freshman	12
sophomore	10
junior	3
senior	5

The probability of selecting a junior **or** a senior is

$$\frac{3 \text{ juniors } + 5 \text{ seniors}}{30 \text{ students}} = \frac{8}{30} \approx 0.2667$$

Class	Frequency
freshman	12
sophomore	10
junior	3
senior	5

Using probability notation, let A be the event we selected a junior and B be the event we selected a senior. Then

$$P(A \text{ or } B) = \frac{8}{30} \approx 0.2667$$