4.3 Rules of Probability

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Goals

1. Use the addition rule to calculate the probability of one event *or* some other event occurring.
2. Use the complement rule to calculate probabilities.

Rules of Probability

Consider a six-sided die.

$$P(\text{roll a 1 or 2}) = \frac{\text{2 ways}}{\text{6 outcomes}} = \frac{1}{3}$$

We get the same result by taking

$$P(\text{roll a 1}) + P(\text{roll a 2}) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

It turns out this is widely applicable!

Addition Rule for Disjoint Outcomes

If A_1 and A_2 are disjoint outcomes, then the probability that one of them occurs is

$$P(A_1 \text{ or } A_2) = P(A_1) + P(A_2)$$

This can also be extended to more than two disjoint outcomes:

$$P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_k) = P(A_1) + P(A_2) + \cdots + P(A_k)$$

for k disjoint outcomes.

Example: A deck of cards.

- Let A be the event that a card drawn is a diamond.
- Let B be the event it is a face card.

A: $2\diamondsuit$ $3\diamondsuit$ $4\diamondsuit$ $5\diamondsuit$ $6\diamondsuit$ $7\diamondsuit$ $8\diamondsuit$ $9\diamondsuit$ $10\diamondsuit$ $J\diamondsuit$ $Q\diamondsuit$ $K\diamondsuit$ $A\diamondsuit$

B: $J\heartsuit$ $Q\heartsuit$ $K\heartsuit$ $J\clubsuit$ $Q\clubsuit$ $K\clubsuit$ $J\diamondsuit$ $Q\diamondsuit$ $K\diamondsuit$ $J\spadesuit$ $Q\spadesuit$ $K\spadesuit$

Example: A deck of cards.

The collection of cards that are diamonds or face cards (or both) is

$A\diamondsuit$ $2\diamondsuit$ $3\diamondsuit$ $4\diamondsuit$ $5\diamondsuit$ $6\diamondsuit$ $7\diamondsuit$ $8\diamondsuit$ $9\diamondsuit$ $10\diamondsuit$ $J\diamondsuit$ $Q\diamondsuit$ $K\diamondsuit$ $J\clubsuit$ $Q\clubsuit$ $K\clubsuit$ $J\heartsuit$ $Q\heartsuit$ $K\heartsuit$ $J\spadesuit$ $Q\spadesuit$ $K\spadesuit$

Looking at these cards, I can see that there are 22 of them, so

$$P(A \text{ or } B) = \frac{22}{52}$$

Example: A deck of cards.

- If I try to apply the addition rule for disjoint outcomes,

$$P(A) = \frac{13}{52}$$

and

$$P(B) = \frac{12}{52}$$

- And we get $\frac{13+15}{52} = \frac{25}{52}$.
- But we know $P(A \text{ or } B) = \frac{22}{52}$.

Example: A deck of cards.

What happened?

- When I added these, I *double counted* $J\diamondsuit$, $Q\diamondsuit$, and $K\diamondsuit$
  - (the cards that are in both A and B).
- I need to subtract off the double count

$$\frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$

General Addition Rule

For any two events A and B, the probability that *at least* one will occur is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

- The general addition rule applies to *any* two events, even disjoint events.
  - For disjoint events, $P(A \text{ and } B) = 0$.

Inclusive or

- When we say "or", we include the situations where:
  - A is true
  - B is true
  - both A and B are true.
- This is an *inclusive or*.

Complements

The **complement** of an event is all of the outcomes in the sample space that are *not* in the event. For an event A, we denote its complement by A^c.

Example

- For a single roll of a six-sided die, the sample space is all possible rolls: 1, 2, 3, 4, 5, or 6.
- Let event A be rolling a 1 or a 2.
- Then A^c is rolling a 3, 4, 5, or 6.

In probability notation:

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{1, 2\}$$

$$A^c = \{3, 4, 5, 6\}$$

Property

$$P(A \text{ or } A^c) = 1$$

- Using the addition rule,

$$P(A \text{ or } A^c) = P(A) + P(A^c) = 1$$

- Make sure you can convince yourself that A and A^c are *always* disjoint.

The Complement Rule

$$P(A) = 1 - P(A^c)$$

Checkpoint

Consider rolling 2 six-sided dice and taking their sum.

The event of interest A is a sum less than 12. Find

1. A^c
2. $P(A^c)$
3. $P(A)$

Hint #1: how many ways can you get a sum of at least 12?

Hint #2: there are 36 possible outcomes.