6.3 Other Levels of Confidence

Dr. Lauren Perry

Goals

1. Use the normal distribution applet to find critical values.
2. Find and interpret confidence intervals for a mean when (A) the population is normal and (B) σ is known.

The 95% confidence interval is common in research, but there's nothing inherently special about it.

- You could calculate a 90%, a 99%, or even something like a 43.8% confidence interval.
- These numbers are called the **confidence level**.
  - They represent the proportion of times that the parameter will fall in the interval (if we took many samples).

Confidence Intervals

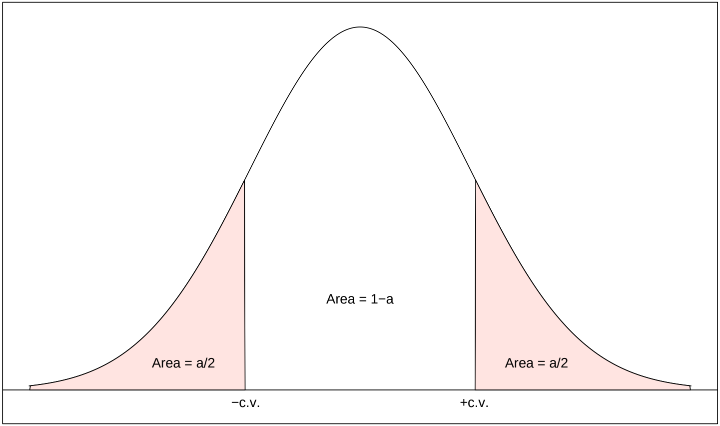
The 100(1-α)% confidence interval for μ is given by

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

where $z_{\alpha/2}$ is the z-score associated with the $[1 - (\alpha/2)]$th percentile of the standard normal distribution.

Critical Values

The value $z_{\alpha/2}$ is called the **critical value** ("c.v." on the plot, below).



Common Critical Values

Confidence Level	α	Critical Value, $z_{\alpha/2}$
90%	0.10	1.645
95%	0.05	1.96
98%	0.02	2.326
99%	0.01	2.575

Example

Prior experience with SAT scores in the CSU system suggests that SAT scores are well-approximated by a normal distribution with standard deviation known to be 50. Suppose we have a random sample of 50 Sac State students with (sample) mean SAT score 1112.

1. Calculate a 98% confidence interval.
2. Interpret the interval in the context of the problem.

Checkpoint

Prior experience with SAT scores in the CSU system suggests that SAT scores are well-approximated by a normal distribution with standard deviation known to be 50. Suppose we have a random sample of 50 Sac State students with (sample) mean SAT score 1112.

3. Calculate a 90% confidence interval.
4. Interpret each interval in the context of the problem. Comment on how the intervals change as you change the confidence level.

Breaking Down a Confidence Interval

$$\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \quad \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

The key values are

- $\bar{x}$, the sample mean
- σ, the population standard deviation
- n, the sample size
- $z_{\alpha/2}$, the critical value

$$P(Z > z_{\alpha/2}) = \frac{\alpha}{2}$$

Breaking Down a Confidence Interval

$$\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \quad \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

- The value of interest is μ, the (unknown) population mean.
- The confidence interval gives us a reasonable range of values for μ.

In addition, the formula includes

- The standard error, $\frac{\sigma}{\sqrt{n}}$
- The margin of error, $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

Confidence Level

$$\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \quad \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

If we can be 99% confident (or even higher), why do we tend to "settle" for 95%??

- What will higher levels of confidence do to this interval?