6.5 Confidence Intervals for a Mean

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Goals

1. Use the t-distribution (applet) to find critical values.
2. Identify when to use the normal distribution and when to use the t-distribution for confidence intervals.
3. Find and interpret confidence intervals for a mean when σ is unknown
   a. When $n \geq 30$
   b. When $n < 30$

Confidence Intervals for a Mean

Realistically, the value of σ is almost never known.

- We know that we can estimate σ using s.
- Can we plug in s for σ? Sometimes!

Central Limit Theorem

- For samples of size $n \geq 30$, $\bar{X}$ will be approximately normal even if X isn't.
- In this case, we can plug in s for σ:

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}.$$

Example

In 2010, a survey collected responses from 1,154 US residents. One of the questions was "After an average workday, about how many hours do you have to relax or pursue activities that you enjoy?". The average time spent relaxing was 3.68 hours, with a standard deviation of 2.6 hours.

Find and interpret a 95% confidence interval for the average time spent relaxing after work.

The T-Distribution

What do we do when $n < 30$?

If

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

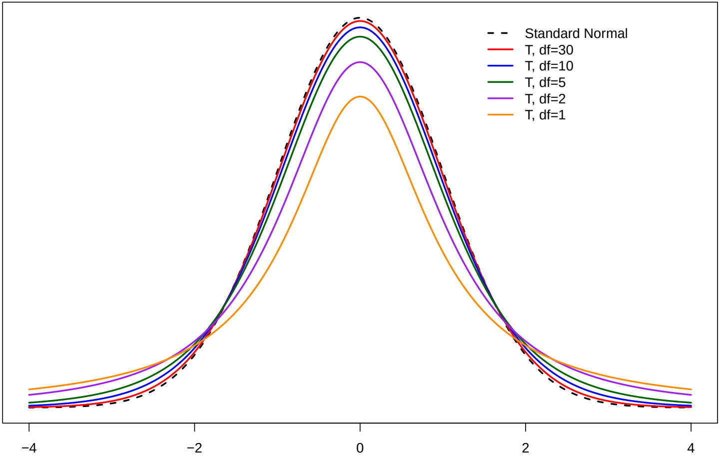
has a standard normal distribution (for X normal or $n \geq 30$), the slightly modified

$$T = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

has what we call the **t-distribution** with $n - 1$ **degrees of freedom**.

The T-Distribution

The only thing we need to know about degrees of freedom is that df $= n - 1$ is the t-distribution's only parameter.



The T-Distribution

- The t-distribution is symmetric and always centered at 0.
- When $n \geq 30$, the t-distribution is approximately equivalent to the standard normal distribution.
- For smaller sample sizes, the t-distribution has more area in the tails.

A General Confidence Interval for the Mean

- Plug in s for σ and use a t critical value $t_{\text{df},\,\alpha/2}$.
  - The t critical value is the $[1-\alpha/2]$th percentile of the t-distribution with $n-1$ degrees of freedom.
- The resulting 95% confidence interval is

$$\bar{x} \pm t_{\text{df},\,\alpha/2} \frac{s}{\sqrt{n}}.$$

T-Distribution Applet

Rossman and Chance t Probability Calculator

- For this applet, enter the degrees of freedom $n-1$ next to "df".
- Then check the top box under "t-value probability" and make sure the inequality is clicked to "$>$" .
- Enter the value of $\alpha/2$ for the probability.
- Click anywhere else on the page and the applet will automatically fill in the box under "t-value".
- This is your t critical value.

Example

Find t critical values for the following settings:

1. A 95% confidence interval with $n = 17$
2. A 90% confidence interval with $n = 8$
3. A 98% confidence interval with $n = 24$

Example

The weights of 6 randomly selected NFL linebackers were

$$243, 238, 229, 253, 248, 225$$

. Find and interpret a 90% confidence interval for the average weight of NFL linebackers.

Checkpoint

A different sample of NFL linebacker weights from 23 randomly selected players resulted in a mean of 242 pounds and a standard deviation of 13 pounds. Based on this new sample, find and interpret a 98% confidence interval for the average weight of NFL linebackers.

Homework Problems

Section 6.5 Exercises 1-4