7.4 The P-Value Approach to Hypothesis Testing

Dr. Lauren Perry

Goals

1. Test one sample means using the p-value approach.
2. Interpret p-values.

If the null hypothesis is true, what is the probability of getting a random sample that is as inconsistent with the null hypothesis as the random sample we got?

- This probability is called the **p-value**.

Idea

- Probability of a sample *as inconsistent* as our sample is

$$P(t_{df} \text{ is as extreme as the test statistic})$$

- Something like $P(t_{18} > 3.6) = 0.001$
  - . . . but we want to think about the probability of being "as extreme" in *either direction*, so

$$\text{p-value} = 2P(t_{18} > 3.6) = 0.002$$

If p-value $< \alpha$, reject the null hypothesis. Otherwise, do not reject.

P-Values

- **Large Sample Setting**: μ is target parameter, $n \geq 30$,

$$2P(Z > |z|)$$

where z is the test statistic.

- **Small Sample Setting**: μ is target parameter, $n < 30$,

$$2P(t_{df} > |t|)$$

where t is the test statistic.

Steps

1. State the null and alternative hypotheses.
2. Determine the significance level α. Check assumptions (decide which setting to use).
3. Compute the value of the test statistic.
4. Determine the p-value.
5. If p-value $< \alpha$, reject the null hypothesis. Otherwise, do not reject.
6. Interpret results.

We often use p-values instead of the critical value approach because they are meaningful on their own (they have a direct interpretation).

Example

Is the average meerkat height different from 30cm? A random sample of 18 meerkats yielded a mean of 26.5cm and a standard deviation of 2.07cm. Use the p-value approach to test at the 0.05 level of significance.

Checkpoint

Is the average number of eggs in a green sea turtle nest different from 120? A random sample of 20 green sea turtle eggs resulted in a mean of 108 eggs with standard deviation 14.48. Use the p-value approach to test at the 0.1 level of significance.