8.1 Confidence Intervals for a Proportion

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Confidence Intervals for a Proportion

Inference for a proportion is really similar to inference for a mean!

- We can apply the Central Limit Theorem to the sampling distribution for a proportion.
  - But... isn't our Central Limit Theorem only for means?

CLT for Binomial?

- A binomial experiment is made up of a series of Bernoulli trials, which result in 0s and 1s.
- If we add up these values, we get the number of successes x.
- If we take the mean of these successes, we get the *proportion* of successes.
- That is, $\bar{x} = \hat{p}$ and we can work with the sampling distribution for a sample mean!

CLT for Binomial

By the Central Limit Theorem, $\hat{p}$ is approximately normally distributed with mean

$$\mu_{\hat{p}} = p$$

and standard error

$$\sigma_{\hat{p}} = \frac{\sqrt{p(1-p)}}{\sqrt{n}} = \sqrt{\frac{p(1-p)}{n}}$$

General Confidence Intervals

- Confidence intervals all use the same basic formula:

$$\text{estimate} \pm \text{critical value} \times \text{standard error}$$

- We do not know the true value of p for the standard error

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

so we will plug in $\hat{p}$.

A $100(1-\alpha)\%$ confidence interval for p:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

To use this formula, we need to check that $n\hat{p} > 10$ and $n(1-\hat{p}) > 10$.

Example

Suppose we take a random sample of 27 US households and find that 15 of them have dogs. Find and interpret a 95% confidence interval for the proportion of US households with dogs.

Checkpoint

Suppose we take a random sample of 45 Sac State students and find that 13 of them are in SSIS. Find and interpret a 95% confidence interval for the proportion of Sac State students in SSIS.