

One-Sided Hypothesis Testing for a Proportion

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Choosing a Significance Level

- The standard significance level for most fields is $\alpha = 0.05$
- Sometimes we may adjust the significance level based on application.
 - For example, if making a Type I error is especially problematic, we might reduce our significance level slightly.
 - If making a Type II error is especially problematic, we might increase our significance level slightly.
- Collecting more data also reduces Type II error, so whenever reasonable it's a good idea to collect more data!

Statistical Significance vs Practical Significance

- For very large sample sizes, point estimates can become very precise.
- In these cases, differences between the point estimate and null value become easier to detect.
- Sometimes we can detect even an incredibly small difference!
- These differences are statistically significant, but may not be practically significant.

Statistical Significance vs Practical Significance

For example,

- An online experiment might identify that placing additional ads on a movie review website statistically significantly increases viewership of a TV show by 0.001%.
- But... who cares about an increase of 0.001%?
- This increase probably has no practical value.

One-Sided Hypotheses

So far we've considered only **two-sided hypotheses**:

$$H_0 : p = p_0$$

$$H_A : p \neq p_0$$

Think of two-sided as when the alternative hypothesis has values *on either side* of the null proportion.

One-Sided Hypotheses

For a **one-sided hypothesis test**, the alternative hypothesis has values on only one side of the null proportion. Either

$$H_A : p > p_0$$

or

$$H_A : p < p_0$$

One-Sided Hypotheses

We ALWAYS use equality in our null hypothesis! By convention, we write

$$H_0 : p \leq p_0$$

$$H_A : p > p_0$$

or

$$H_0 : p \geq p_0$$

$$H_A : p < p_0$$

One-Sided Hypothesis Testing

There is only one major difference in one-sided hypothesis testing.

- For the test statistic approach
 - We use the critical value z_α instead of $z_{\alpha/2}$.
- For the p-value approach
 - We no longer multiply by two: $\text{p-value} = P(|Z| > |ts|)$

One-Sided Hypothesis Testing

In both cases,

- We are no longer interested in seeing observations as extreme as \hat{p} .
- Now we are actively interested in a particular direction corresponding to the direction of the alternative hypothesis.
- This means that we are interested in a particular Z -score or a single tail area.

One-Sided Hypothesis Testing

This is useful for several reasons

- ① We don't have to find $z_{\alpha/2}$ or double the p-value, so the level of evidence required to reject H_0 goes down.
- ② Sometimes we are really only interested in one direction.

On the flip side, we lose the ability to detect any interesting findings in the opposite direction.

Example

In our first lecture, we talked through an example where doctors were interested in determining whether stents would help people who had a high risk of stroke.

- The researchers believed the stents would help.
- The data suggested the opposite, that stents were actively harmful.

A one-sided test could have checked whether the stents were helpful, but a two-sided test allowed the researchers to see that there was harm being done.

One-Sided Hypotheses

- Using one-sided hypotheses runs the risk of overlooking data supporting an opposite conclusion.
- We might have made the same error had we run a one-sided test on the world health question data.
 - We probably would have been tempted to test

$$H_0 : p \leq 0.333$$

$$H_A : p > 0.333$$

to see if people were better than chance.

- ...and we would have missed the finding that people actually do worse than chance levels!

One-Sided Hypothesis Tests

So when should you use a one-sided test? Rarely! Before using a one-sided test, consider

- What would we conclude if the data happens to go clearly in the opposite direction?
- Is there any value in learning about the data doing in the opposite direction?

If there is any value in this, use a two-sided test!

Why Can't We Look at the Data First?

We should always always always *always* set up our hypotheses and analysis plan *before taking any data!*

- This is part of doing good science!
- If we pick hypotheses after seeing the data,
 - If $\hat{p} < p_0$, and we set $H_A: p < p_0$, any observation in the lower 5% of the null distribution would lead to us rejecting H_0 .
 - If $\hat{p} > p_0$, and we set $H_A: p < p_0$, any observation in the upper 5% of the null distribution would lead to us rejecting H_0 .

Under H_0 , we now have a $0.05 + 0.05 = 0.1$ probability of Type I error!

A Note About Textbook Sections

We've covered everything in section 6.1