Hypothesis Tests for Two-Sample Means

October 7, 2019

Difference of Two Means

What happens when we have two population means, but the data are not paired?

- The approach is similar to paired data.
- We will need a little bit more detail about each sample.
- We will also develop a new standard error formula.

Examples

- Do stem cells improve heart function?
- Is there a relationship between a pregnant person's smoking habits and birth weight?
- Is one version of an exam harder than another?

Confidence Interval for Difference of Means

Research question: Does treatment using embryonic stem cells (ESCs) help improve heart function following a heart attack?

- Tested in sheep post heart attack.
- 9 sheep assigned to treatment group (ESCs)
- 9 sheep assigned to control (no ESCs)
- Measured change in hearts' pumping capacity.

Confidence Intervals for Difference of Means

Summary statistics:

	n	$\bar{x}$	s
ESCs	9	3.50	5.17
control	9	-4.33	2.76

The point estimate for the difference of population means is the difference of the sample means.

The t-Distribution for Difference of Means

To use a t-distribution, we require

1. Independence
   - Within groups.
   - Between groups
2. Normality
   - Check each group separately.

The t-Distribution for Difference of Means

The standard error may now be computed as

$$SE = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

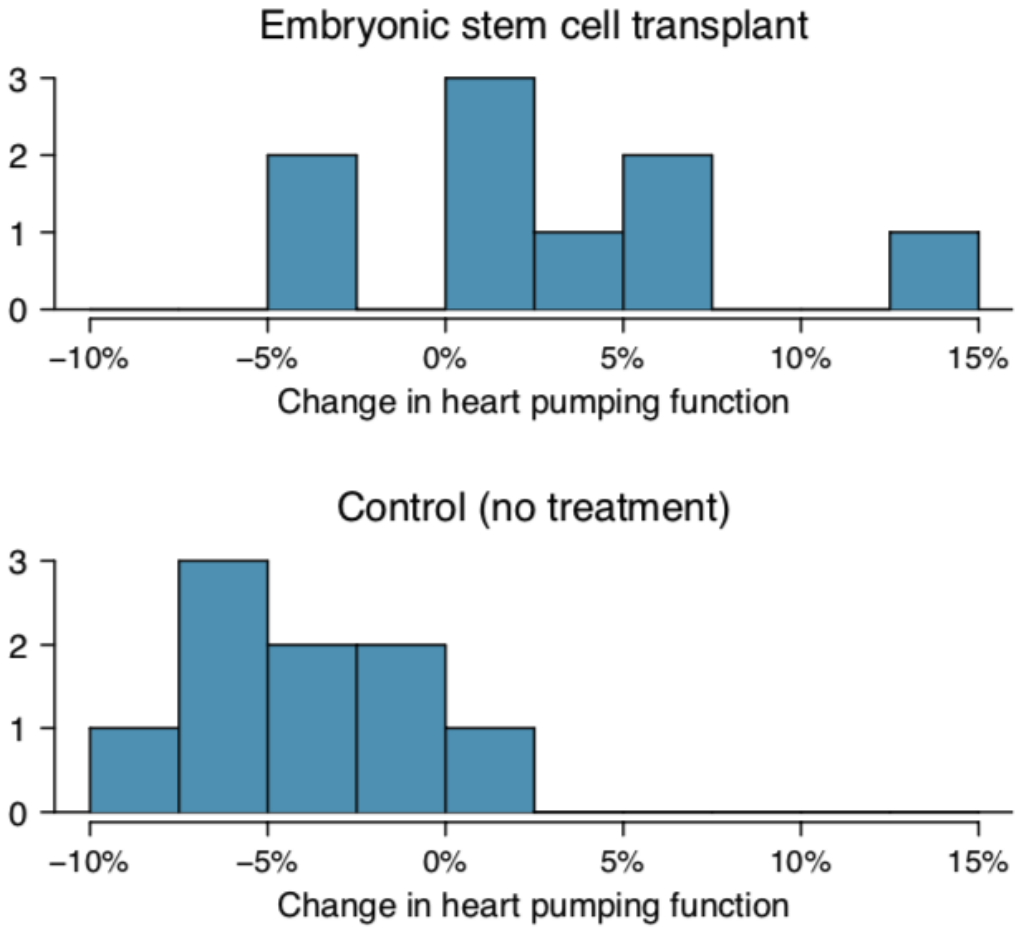
The degrees of freedom is calculated using a complex formula, but for this course you may use

$$\min(n_1 - 1, n_2 - 1)$$

Example: Sheep and ESCs

Can we use the t-distribution for inference about the point estimate $\bar{x}_{esc} - \bar{x}_{control} = 7.83$?

Example: Sheep and ESCs



Example: Sheep and ESCs

Calculate the standard error and degrees of freedom for the ESC research using the summary statistics:

	n	$\bar{x}$	s
ESCs	9	3.50	5.17
control	9	-4.33	2.76

Example: Sheep and ESCs

Calculate a 95% confidence interval for the difference in heart pumping capacity between ESCs and the control.

Statistical Inference Procedure

The details may change, but the general approach is always:

1. **Prepare.** Pick out critical contextual information and set up hypotheses.
2. **Check** conditions.
3. **Calculate** standard error and confidence interval/test statistic.
4. **Conclude** based on context.

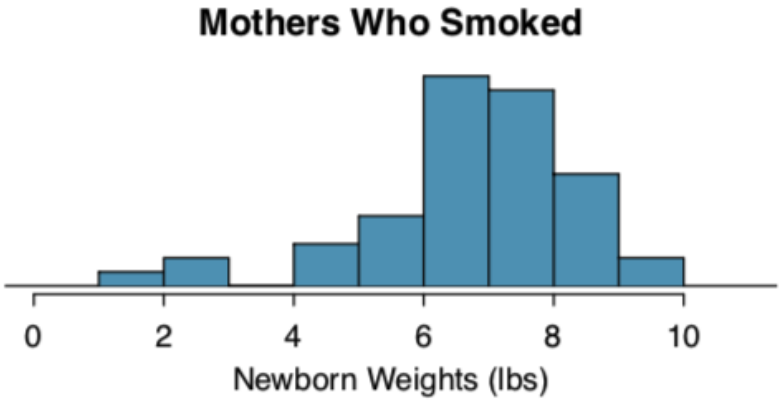
Hypothesis Tests for Difference of Two Means

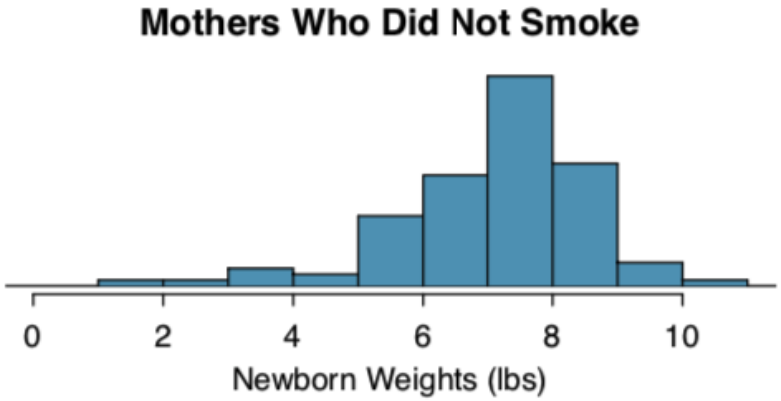
North Carolina births data:

- 150 mothers with newborns
- `weight`: weight of newborn
- `smoke`: mother's smoking habits during pregnancy (yes/no)

Question: Do newborns from mothers who smoke have a different average birth weight than those from mothers who don't smoke?

North Carolina Births Data





	smoker	nonsmoker
mean	6.78	7.18
std dev	1.43	1.60
sample size	50	100

North Carolina Births Data

- Set up hypotheses for the birth weight and smoking data.

North Carolina Births Data

- Set up hypotheses for the birth weight and smoking data.
- Check conditions.

North Carolina Births Data

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- Check conditions.
- Calculate the point estimate and standard error.

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- Set up hypotheses for the birth weight and smoking data.
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North Carolina Births Data

- Set up hypotheses for the birth weight and smoking data.
- Check conditions.
- Calculate the point estimate and standard error.
- Find the test statistic.
- Find the critical value.
- Draw conclusions.

North Carolina Births Data

The overall scientific conclusion is that smoking results in a lower birth weight. What happened?