Introduction to Experimental Design

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Experimental Design

You know a lot about experimental design already, but we want to contextualize some of those concepts.

Observational Studies

For an **observational study**,

- A **sampling plan** refers to the way a sample is selected.
- Data is observed rather than produced.
- Example: sample surveys.

Experiments

For an **experiment**,

- **Experimental design** refers to the way a sample is selected.
- One or more conditions are imposed by the researcher.

Experimental Design Terms

- An **experimental unit** is the object on which a measurement is taken.
- A **factor** is an independent variable whose values are controlled/varied by the experimenter.
- A **level** is the intensity (value) of a factor.
- A **treatment** is a specific combination of factor levels.
- The **response** is the variable being measured.

Example

- A group of people is randomly divided into an experimental and a control group.
- The control group is given an aptitude test after having eaten a full breakfast.
- The experimental group is given the same test without having eaten any breakfast.

What are the factors, levels, and treatments in this experiment?

Example

- Suppose that the experimenter in the previous example began by randomly selecting 20 men and 20 women for the experiment.
- These two groups were then randomly divided into 10 each for the experimental and control groups.

What are the factors, levels, and treatments in this experiment?

The Completely Randomized Design

- Random samples are selected independently from each of k populations.
- There is only one factor, so this is called a **one-way classification**.
- Sound familiar? We already know how to analyze this data!
- We can analyze completely randomized designs using t-tests ($k = 1$ or 2) or ANOVA ($k > 2$).

The Randomized Block Design

- The completely randomized design is best used when experimental units are *homogeneous* (the same/similar).
- It also allows us to examine only one factor (the treatment or groups).
- Any other variability in the response gets lumped in with experimental error.

The Randomized Block Design

- Sometimes units are not at all homogeneous.
- Typically we aren't interested in this source of variation.
- Instead, we want to control for it.
- E.g., if I'm looking at the impact of growth hormones on rats, I may want to control for differences between males and females.

The Randomized Block Design

We isolate this additional information using a **randomized block design**.

- We are still interested in comparing k treatment means.
- Now we will also have b blocks.
- Each block should be made up of homogeneous experimental units.
- We will have $n = b \times k$ observations.

The ANOVA for Randomized Block Designs

- We now have two factors: treatments/groups and blocks.
- Each will affect the response.
- This is sometimes called a two-way ANOVA.

Sum of Squares for Randomized Block Designs

The total sum of squares is now partitioned into three sources of variation:

$$\text{SSTotal} = \text{SSG} + \text{SSB} + \text{SSE}$$

- SSG: sum of squares, groups
- SSB: sum of squares, blocks
- SSE: sum of squares, error

Degrees of Freedom for Randomized Block Design

Each source of variation has an accompanying degrees of freedom:

- $df_{\text{groups}} = k - 1$
- $df_{\text{blocks}} = b - 1$
- $df_{\text{error}} = (k - 1)(b - 1)$
- $df_{\text{total}} = n - 1 = b \times k - 1$

The mean square for each source of variation is the sum of squares divided by its degrees of freedom.

ANOVA: Randomized Block Design

Source	df	SS	MS	F
Group	$k-1$	SSG	$MSG = \frac{SSG}{k-1}$	MSG/MSE
Blocks	$b-1$	SSB	$MSG = \frac{SSG}{b-1}$	MSG/MSE
Error	$(k-1)(b-1)$	SSE	$MSE = \frac{SSE}{(k-1)(b-1)}$	
Total	$n-1 = bk-1$	SST		

Example

- The cost of a cellphone minute varies drastically depending on the number of minutes per month used by the customer.
- A consumer watchdog group decided to compare the average costs for four cellular phone companies using three different usage levels as blocks.
- The monthly costs (in dollars) were computed for peak-time callers at low (20 minutes per month), middle (150 minutes per month), and high (1000 minutes per month).
- We want to construct the analysis of variance table for this experiment.

Example

The data is shown below.

	Company				
Usage Level	A	B	C	D	Total
Low	27	24	31	23	105
Middle	68	76	65	67	276
High	308	326	213	300	1246
Total	403	426	408	390	1627

Using this data, we can use a computer to find SSTotal$= 189,798.9167$, SSGroup$= 222.25$, and SSBlock$= 189,335.1667$.

Hypothesis Tests

Now we have two F values to worry about. These correspond to tests regarding treatment means

$$H_0: \quad \text{No difference among } k \text{ group means.}$$
$$H_A: \quad \text{At least one pair of group means is not equal.}$$

and block means

$$H_0: \quad \text{No difference among } b \text{ block means.}$$
$$H_A: \quad \text{At least one pair of block means is not equal.}$$

Two-Way ANOVA

The intuition behind partitioning variance and the approach to the F-tests are exactly the same as with the one-way ANOVA that we learned last week!

Example

1. Write the hypotheses for the ANOVA from the previous example.
2. What can we conclude?