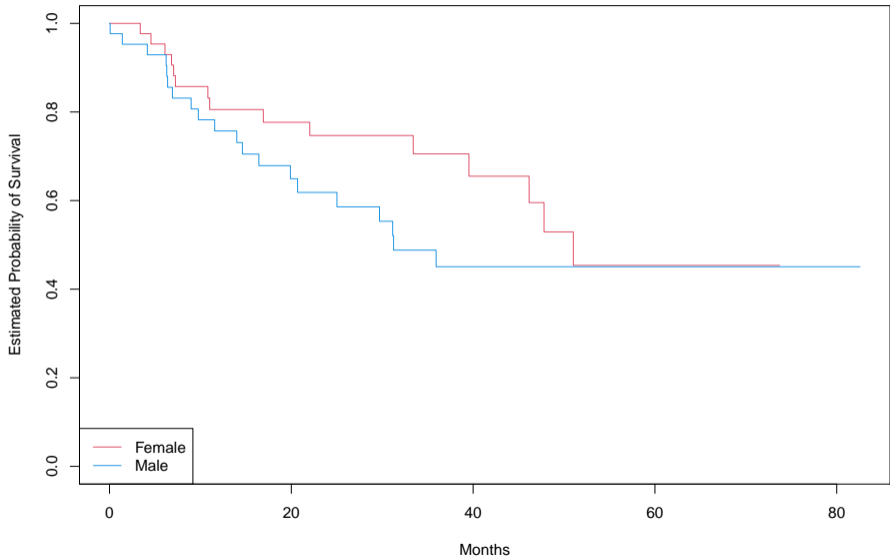


## 11.4 The Log-Rank Test



# The Log-Rank Test

We would like to compare the survival of males to that of females.

Based on the plot,

- ▶ Females seem to fare a little better up to about 50 months, where the two curves both level off at about 50%.
- ▶ How can we carry out a formal test of equality for the two survival curves?
  - ▶ Censoring complicates things like a t-test.

# The Log-Rank Test

- ▶ Recall  $d_1 < d_2 < \dots < d_K$  are the unique death times among the non-censored patients,  $q_k$  is the number of patients who died at time  $d_k$ ,  $r_k$  is the number of patients alive and in the study just before  $d_k$ .
- ▶ We further define  $r_{1k}$  and  $r_{2k}$  to be the number of patients in groups 1 and 2, respectively, who are at risk at time  $d_k$ .
- ▶ We define  $q_{1k}$  and  $q_{2k}$  similarly

## Details of the Test Statistic

	Group 1	Group 2	Total
Died	$q_{1k}$	$q_{2k}$	$q_k$
Survived	$r_{1k} - q_{1k}$	$r_{2k} - q_{2k}$	$r_k - q_k$
Total	$r_{1k}$	$r_{2k}$	$r_k$

At each death time  $d_k$ , we construct a  $2 \times 2$  table of counts.

## Log Rank Test: Main Idea

To test  $H_0 : E(X) = 0$  for some random variable  $X$ , we can construct a test statistic of the form

$$W = \frac{X - E(X)}{\sqrt{\text{Var}(X)}}$$

In order to construct the log-rank test statistic, we compute a quantity that takes the form above, with  $X = \sum_{k=1}^K q_{1k}$

## The Result

The resulting formula for the log-rank test statistic is

$$W = \frac{\sum_{k=1}^K (q_{1k} - E(q_{1k}))}{\sqrt{\sum_{k=1}^K \text{Var}(q_{1k})}} = \frac{\sum_{k=1}^K \left( q_{1k} - \frac{q_k}{r_k} r_{1k} \right)}{\sqrt{\sum_{k=1}^K \frac{q_k (r_{1k}/r_k) (1 - r_{1k}/r_k) (r_k - q_k)}{r_k - 1}}}$$

when the sample is large enough,  $W$  is approximately standard normal.

## Example: Brain Cancer

```
logrank.test <- survdiff(Surv(time, status) ~ sex)
```

```
logrank.test
```

```
## Call:
```

```
## survdiff(formula = Surv(time, status) ~ sex)
```

```
##
```

```
##           N Observed Expected (O-E)^2/E (O-E)^2/V
```

```
## sex=Female 45         15      18.5      0.676      1.44
```

```
## sex=Male   43         20      16.5      0.761      1.44
```

```
##
```

```
## Chisq= 1.4  on 1 degrees of freedom, p= 0.2
```

## Example: Brain Cancer

- ▶ Comparing the survival times of females and males on the `BrainCancer` data gives a log-rank test statistic of  $W = 1.2$ , which corresponds to a two-sided  $p$ -value of 0.2.
- ▶ Thus, we cannot reject the null hypothesis of no difference in survival curves between females and males.
- ▶ The log-rank test is closely related to Cox's proportional hazards model, which we discuss in the next section.